SINGLE CLAUSE ASSUMPTION WITHOUT ACTIVATION LITERALS TO SPEED-UP IC3

FMCAD'21

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You will see

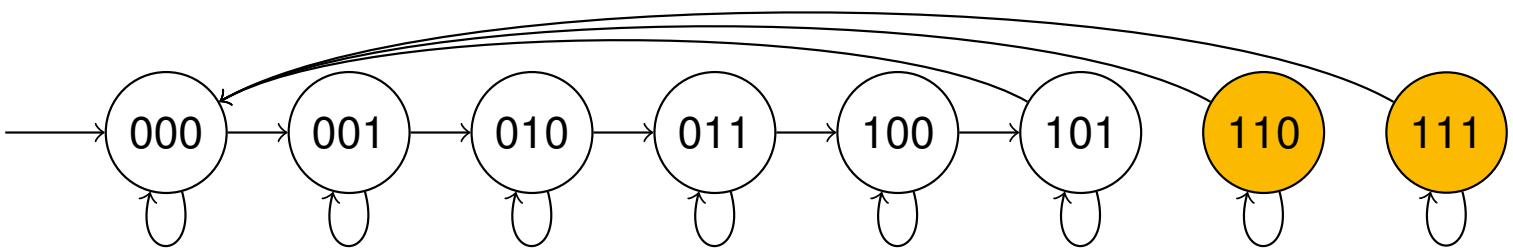
- Extension to the assumption based interface
- Allowing the addition of a single temporary clause
- Optimizing incremental SAT solvers for IC3

$$
\begin{aligned}
&F_0 \\
&F_1 = F_0 \wedge C_1 \wedge C_2 \\
&F_2 = F_1 \wedge C_3 \\
&\vdots
\end{aligned}
$$

$$
F_i \wedge \underbrace{\ell \wedge \overline{\ell'}}_{\text{Assumptions}} \wedge \underbrace{C}_{\text{Constraint}}
$$

Introduction

- 3-bit ($b_2 b_1 b_0$) counter
- Property: Counter does not exceed 5



Introduction

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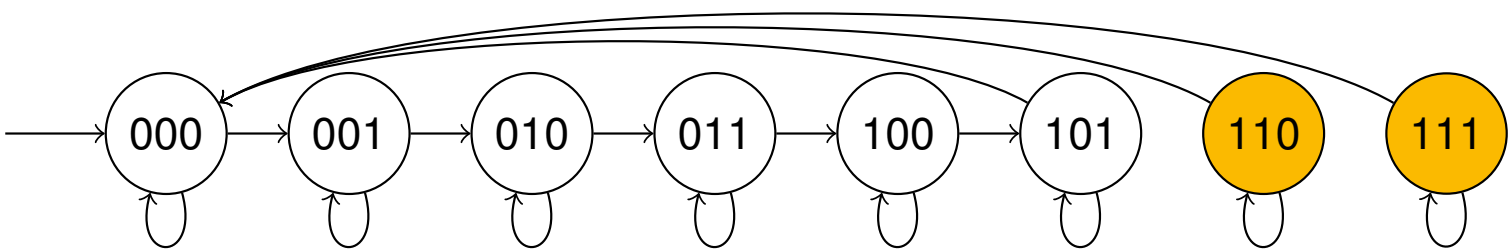
- Property: Counter does not exceed 5
- $T = \quad (\overline{b_2} \wedge \overline{b_1} \wedge \overline{b_0}) \Rightarrow ((\overline{b_2'} \wedge \overline{b_1'} \wedge \overline{b_0'}) \vee (\overline{b_2'} \wedge \overline{b_1'} \wedge \overline{b_0'}))$
  $\wedge (\overline{b_2} \wedge \overline{b_1} \wedge b_0) \Rightarrow ((\overline{b_2'} \wedge \overline{b_1'} \wedge b_0') \vee (\overline{b_2'} \wedge b_1' \wedge \overline{b_0'}))$
  $\wedge \ldots$
- Can state 6 be reached?

$$T \wedge b_2' \wedge b_1' \wedge \overline{b_0'} \quad \text{SAT}$$



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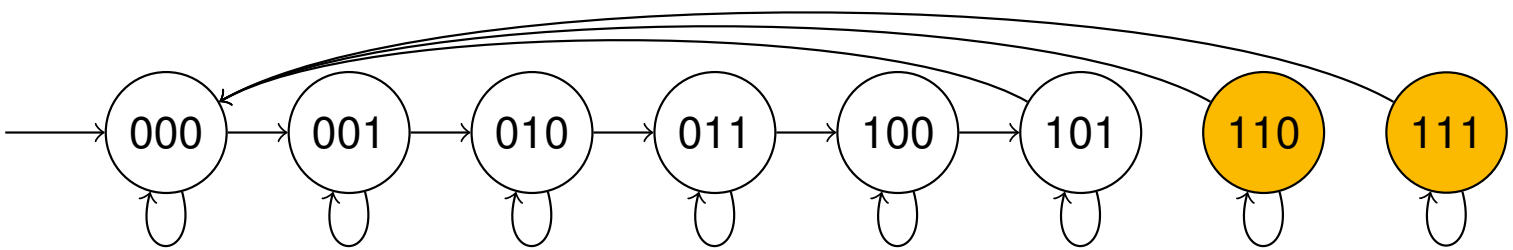
  $\wedge \ldots$
- Can state 6 be reached?

$(\overline{b_2} \vee \overline{b_1} \vee b_0) \wedge T \wedge b'_2 \wedge b'_1 \wedge \overline{b'_0}$ UNSAT $\Rightarrow T \wedge (\overline{b'_2} \vee \overline{b'_1} \vee b'_0)$



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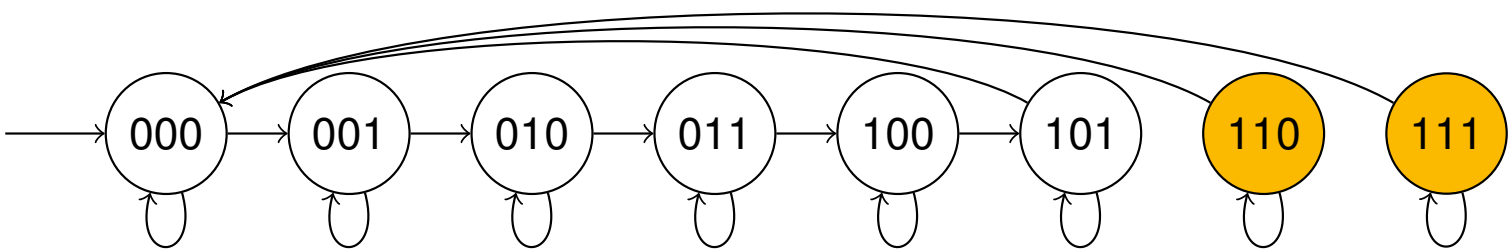
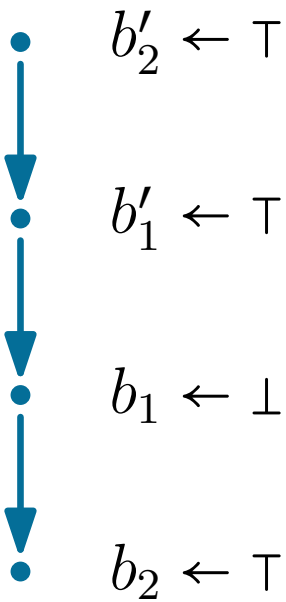
  $\wedge \ldots$
- Can state 7 be reached?

$$(\overline{b_2} \vee \overline{b_1} \vee \overline{b_0}) \wedge T \wedge b_2' \wedge b_1' \wedge b_0' \quad ?$$



Conflict Driven Clause Learning



$b_2' \leftarrow \top$

$b_1' \leftarrow \top$

$b_1 \leftarrow \bot$

$b_2 \leftarrow \top$

C_1 $\overline{b_2} \vee b_1 \vee b_0 \vee \overline{b_1'}$

C_2 $b_2 \vee b_1 \vee \overline{b_0} \vee \overline{b_2'}$

C_3 $\overline{b_2} \vee b_1 \vee \overline{b_0} \vee \overline{b_2'} \vee \overline{b_1'}$

C_4 $b_2 \vee b_1 \vee b_0 \vee \overline{b_2'} \vee \overline{b_0'}$

C_5 $b_2 \vee b_1 \vee b_0 \vee \overline{b_1'} \vee b_0'$

$\ldots$

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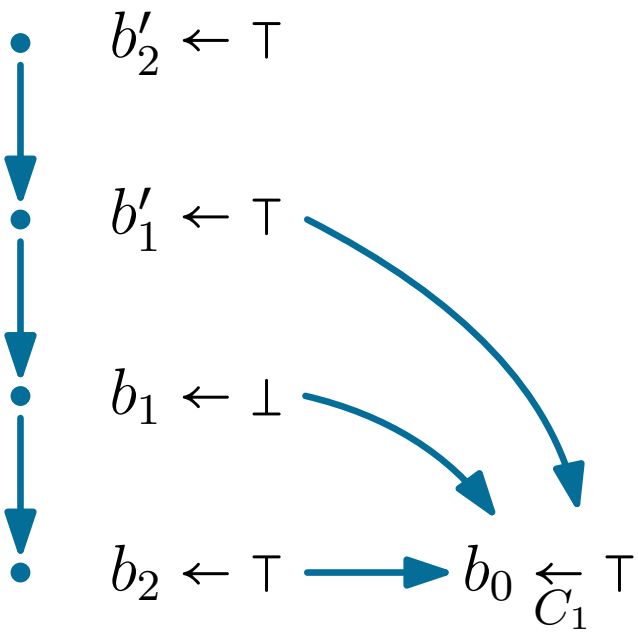

C_1 $\overline{b_2} \vee b_1 \vee b_0 \vee \overline{b'_1}$
C_2 $b_2 \vee b_1 \vee \overline{b_0} \vee \overline{b'_2}$
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C_4 $b_2 \vee b_1 \vee b_0 \vee \overline{b'_2} \vee \overline{b'_0}$
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$\ldots$

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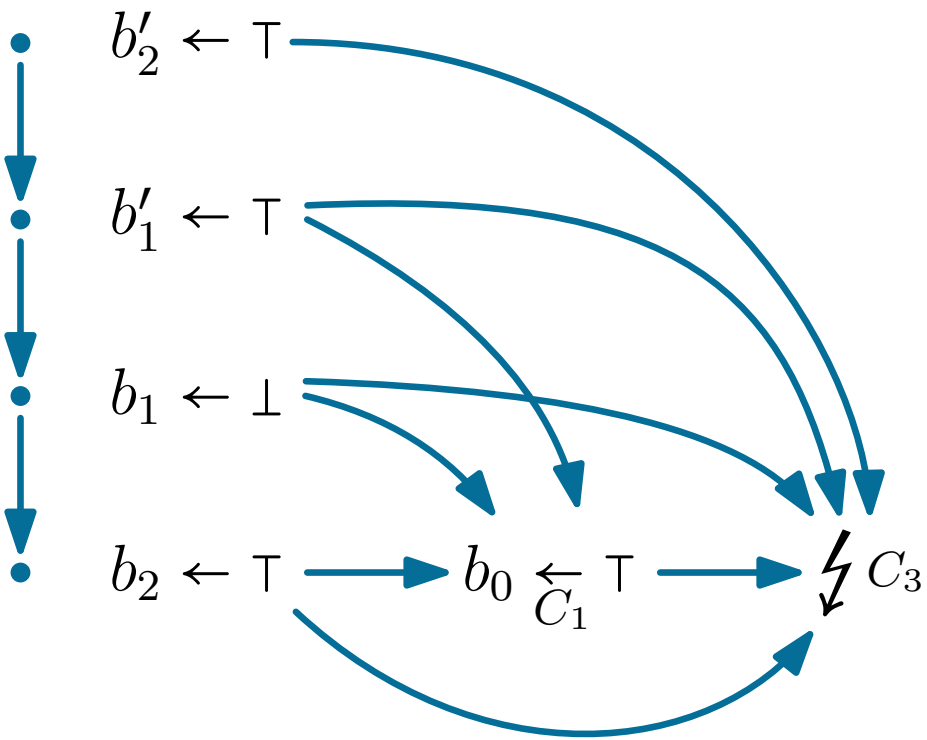

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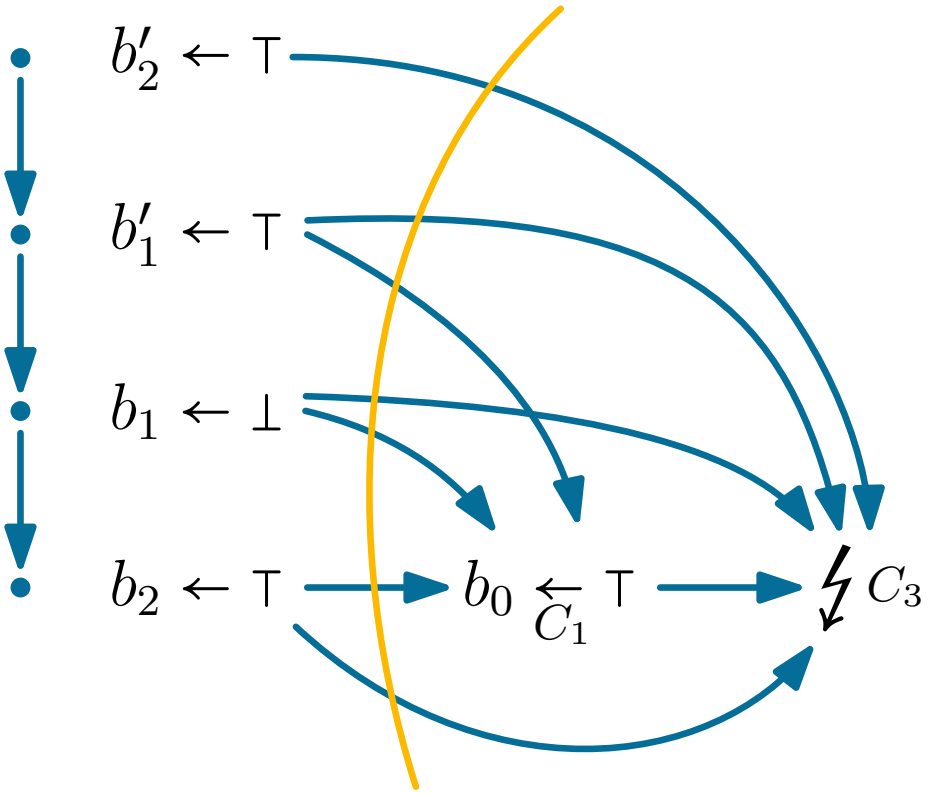

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C_6 $\overline{b_2'} \vee \overline{b_1'} \vee b_1 \vee \overline{b_2}$
$\ldots$

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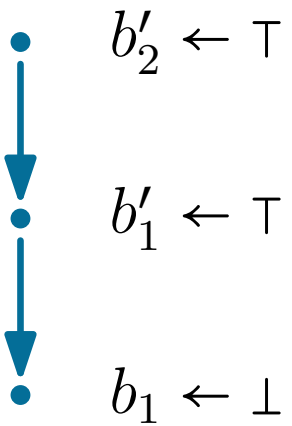

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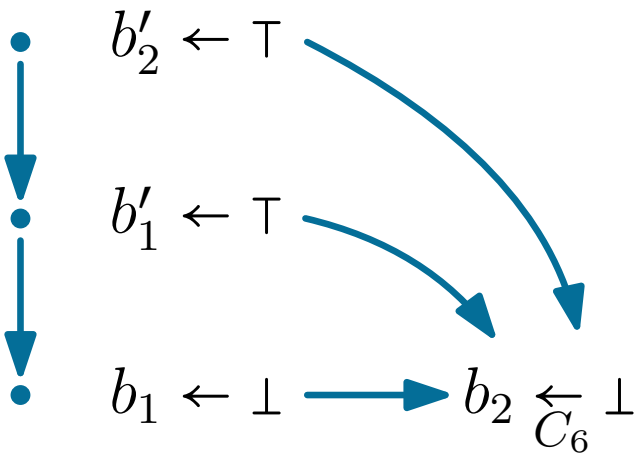

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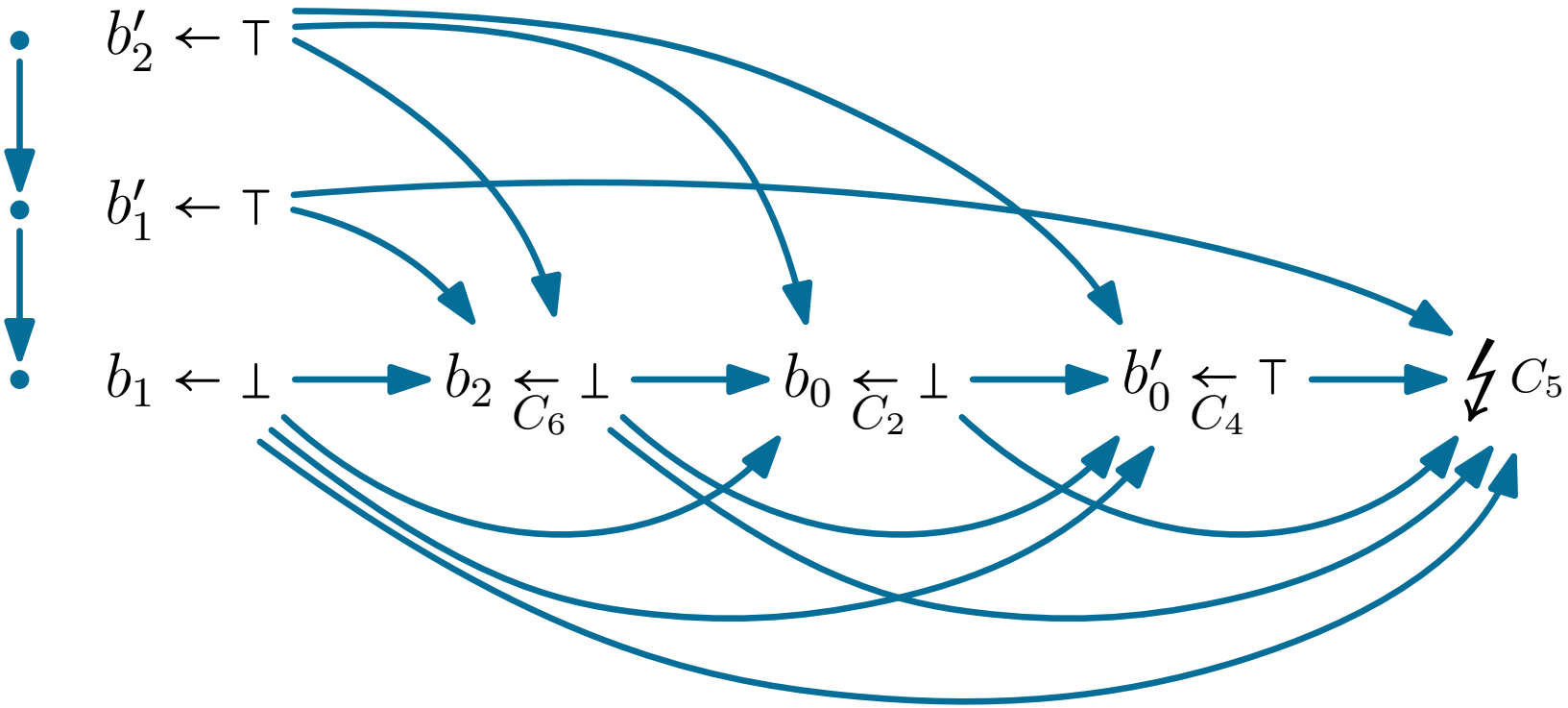

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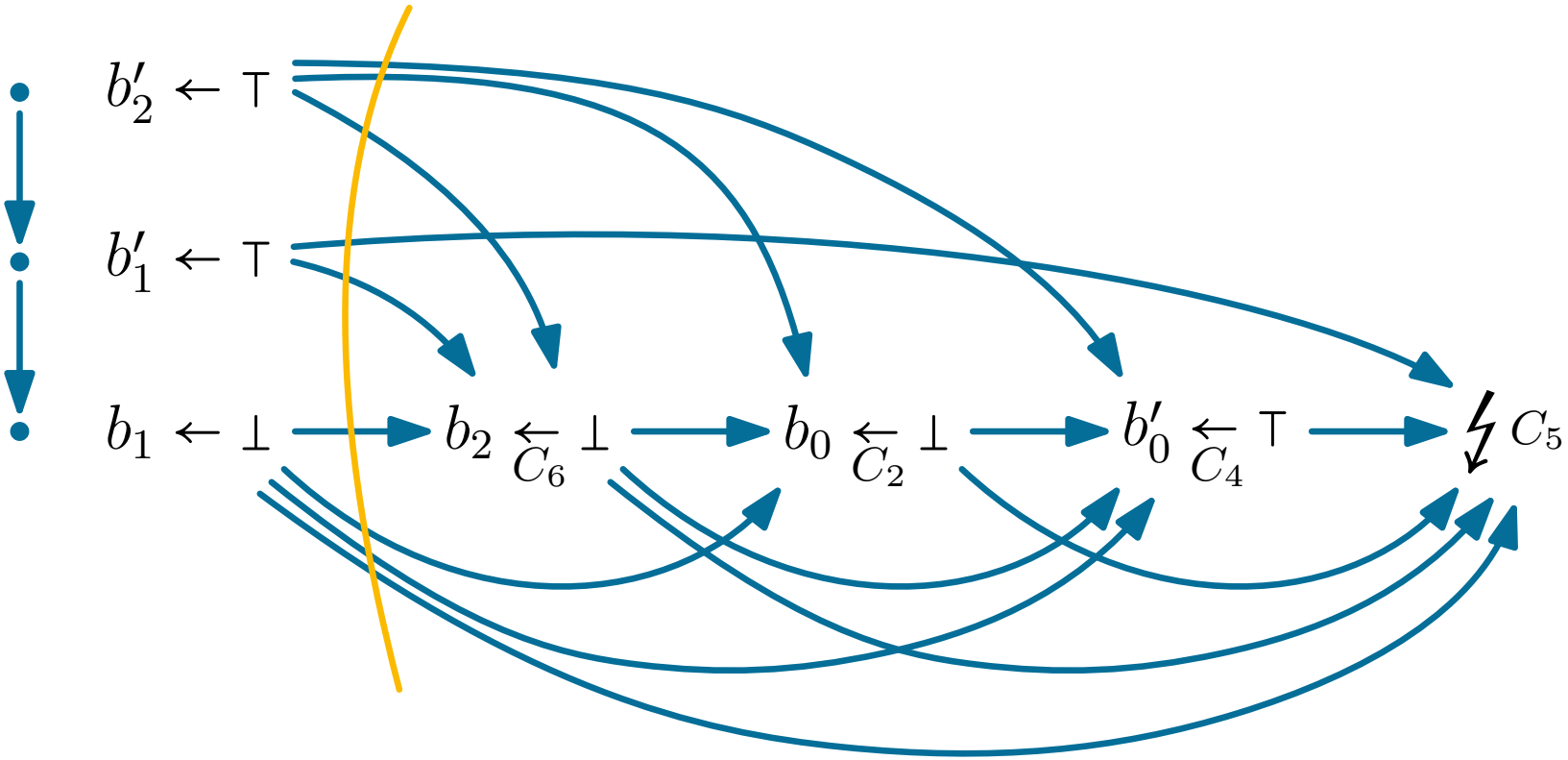

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C_7 $\overline{b'_2} \vee \overline{b'_1} \vee b_1$
$\ldots$

Counter Example

- Three-bit ($b_2 b_1 b_0$) counter
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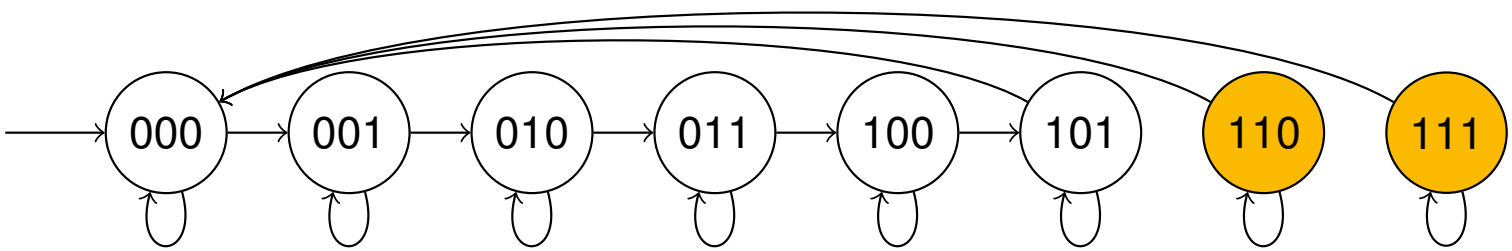
  $\wedge \ldots$
- Can state 7 be reached?

  $(\overline{b_2} \vee \overline{b_1} \vee \overline{b_0}) \wedge T \wedge b_2' \wedge b_1' \wedge b_0'$?



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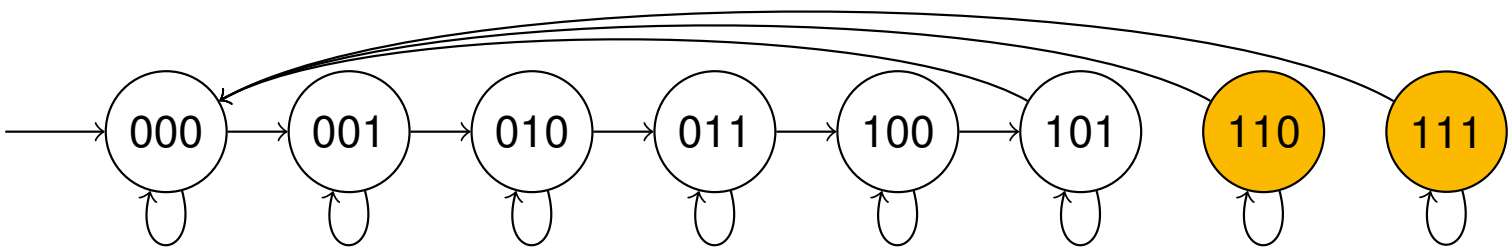
  $\wedge \ldots$
- Can the cube be reached?

110	$(\overline{b_2} \vee \overline{b_1} \vee b_0) \wedge T \wedge b_2' \wedge b_1' \wedge \overline{b_0'}$	UNSAT
11_	$(\overline{b_2} \vee \overline{b_1}) \quad \wedge T \wedge b_2' \wedge b_1'$	UNSAT



Assumptions

$$\underbrace{(\overline{b_2} \vee \overline{b_1} \vee b_0)}_{\text{Constraint}} \quad \wedge \quad T \quad \wedge \quad \underbrace{b_2' \wedge b_1' \wedge \overline{b_0'}}_{\text{Assumptions}}$$

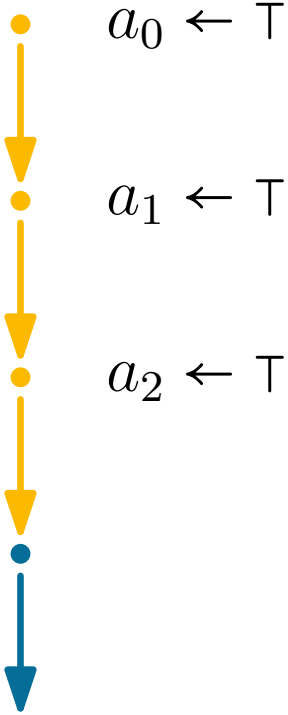
addClause	(`Clause` c)
solve	(`list` < `Literal` > assumptions)

"[. . .] we propose the following interface [. . .] very simple to implement [. . .] expressive enough to encompass several interesting incremental SAT-problems [. . .]"

Temporal Induction by Incremental SAT Solving

[Eén, Niklas and Sörensson, Niklas, 2003]

Assumptions



DECIDE ( )

```
1  if level < |assumptions|
2      ℓ = assumptions[level]
3      if val(ℓ) = false analyzeFinal(ℓ)
4      else trail[level++] = ℓ

5  else trail[level++] = heuristic()
```

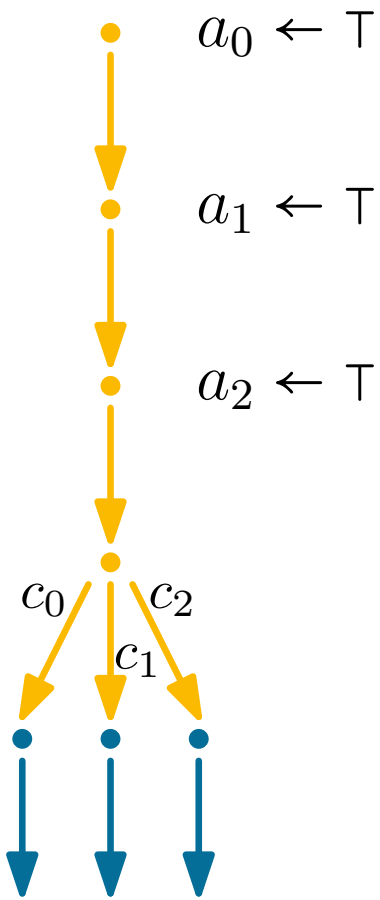
Our Contribution

$$\underbrace{(\overline{b_2} \vee \overline{b_1} \vee b_0)}_{\text{Constraint}} \quad \wedge \quad T \quad \wedge \quad \underbrace{b_2' \wedge b_1' \wedge \overline{b_0'}}_{\text{Assumptions}}$$

addClause	(`Clause` c)
solve	(`list < Literal >` assumptions, `Clause` c)

- Easy to implement
- Expressive enough for IC3
- Learned clauses are save to keep

Constraint

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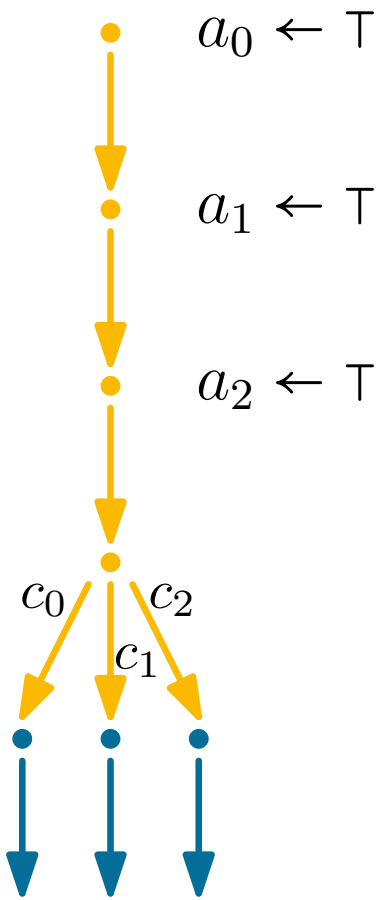
4      else trail[level++] = ℓ
5  else if level = |assumptions|
6      unassignedLit = 0
7      for ℓ in constraint
8          if val(ℓ) = true level++
9          else if val(ℓ) = unassigned
10             unassigendLit = ℓ
11     if unassigendLit = 0
12         analyzeFinal(constraint)
13     else trail[level++] = unassigendLit
14 else trail[level++] = heuristic()
```

Constraint

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Analyze Final

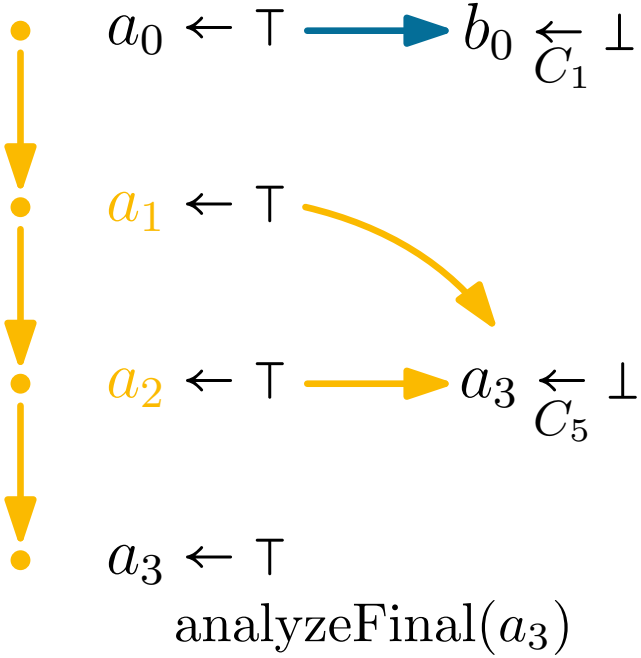
failed(`Literal` ℓ)

Queries if ℓ was used to proof unsatisfiability

If only b_2' and b_1' failed the second cube is free

110	$(\overline{b_2} \vee \overline{b_1} \vee b_0) \wedge T \wedge b_2' \wedge b_1' \wedge \overline{b_0'}$	UNSAT
		$\Downarrow$
11_	$(\overline{b_2} \vee \overline{b_1}) \wedge T \wedge b_2' \wedge b_1'$	UNSAT

Analyze Final

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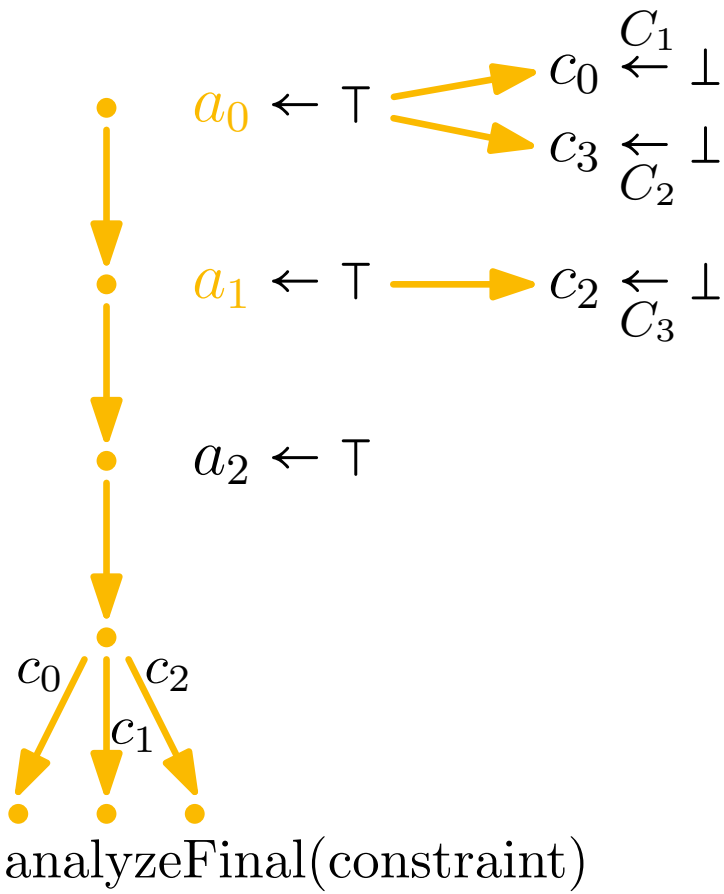


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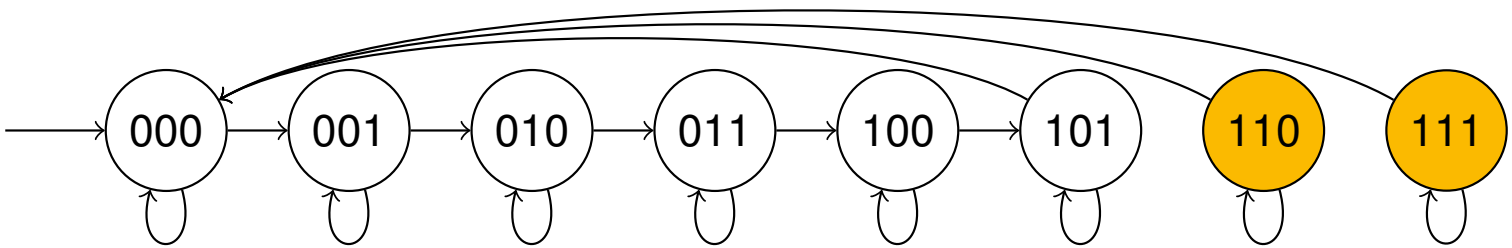
IC3

$$\overline{s} \wedge \quad T \quad \wedge s'$$



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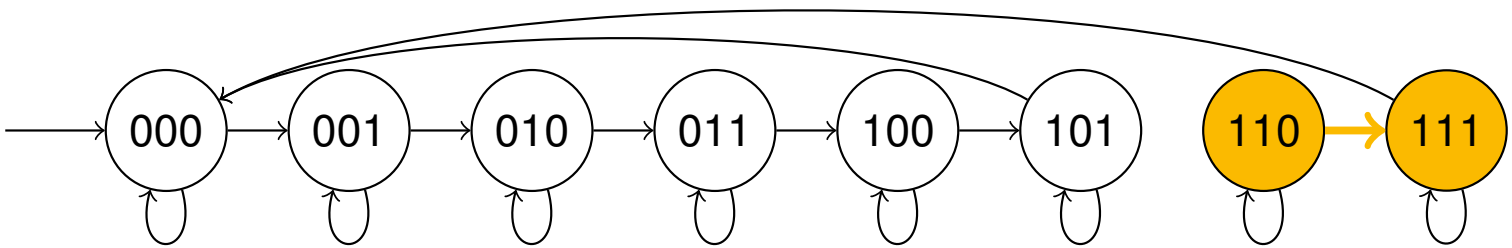

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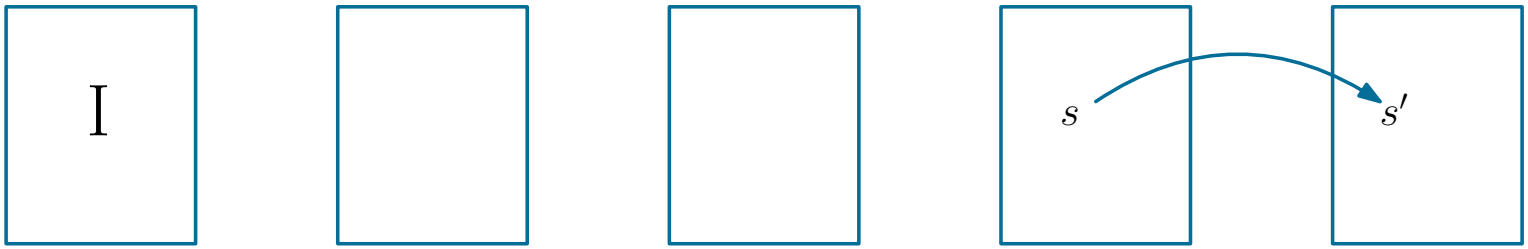
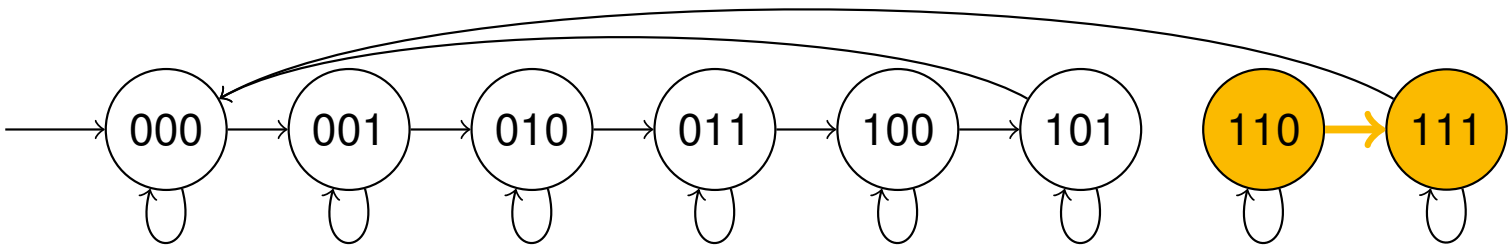


$$\underbrace{\overline{s}}_{\text{Constraint}} \wedge \underbrace{Frame_{i-1} \wedge T}_{\text{Clauses}} \wedge \underbrace{s'}_{\text{Assumptions}}$$



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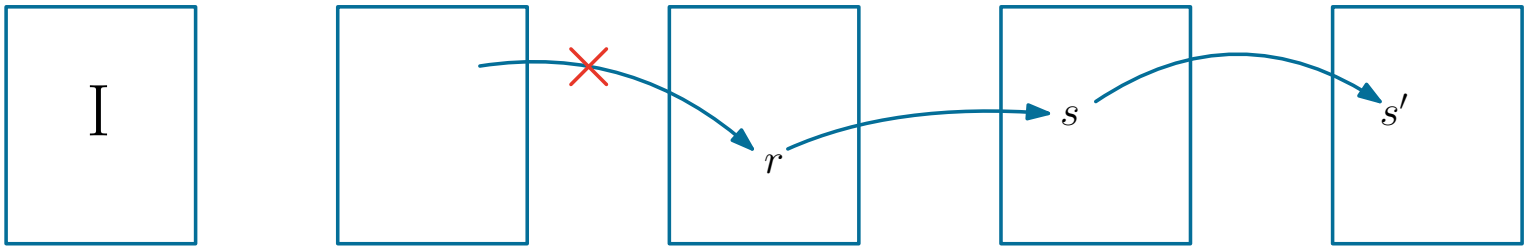

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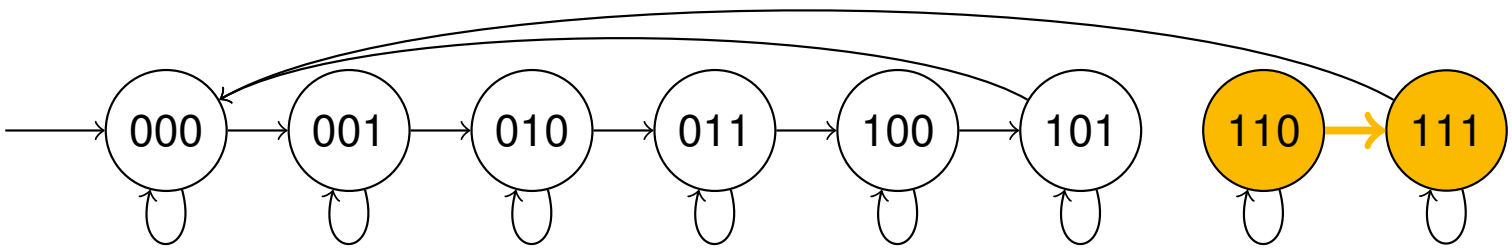
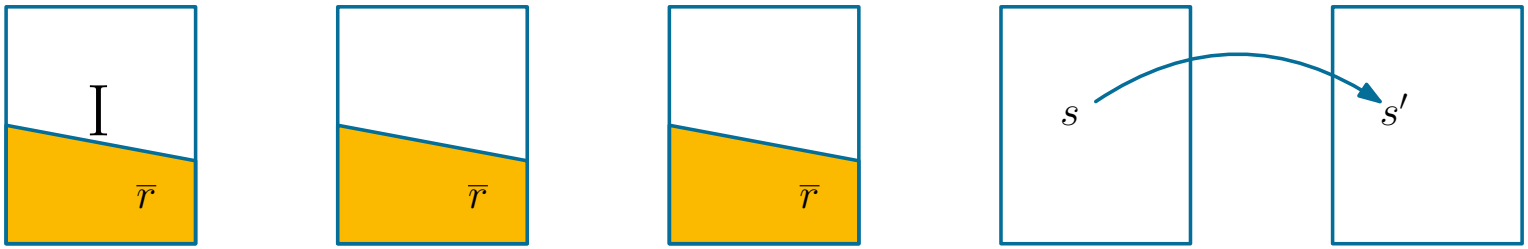
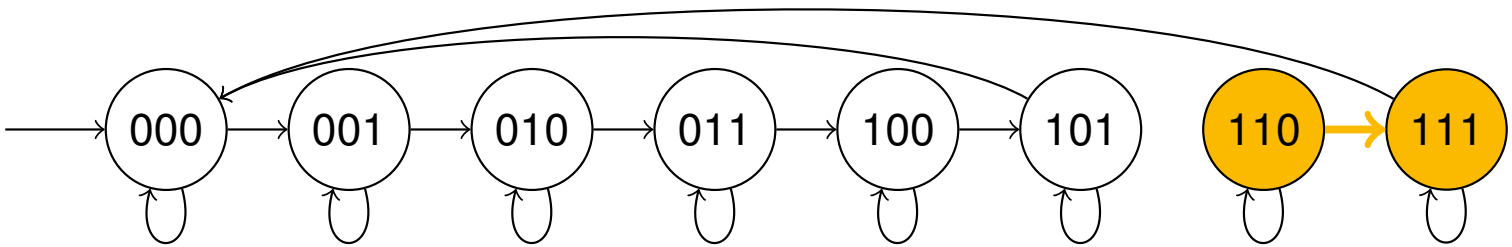


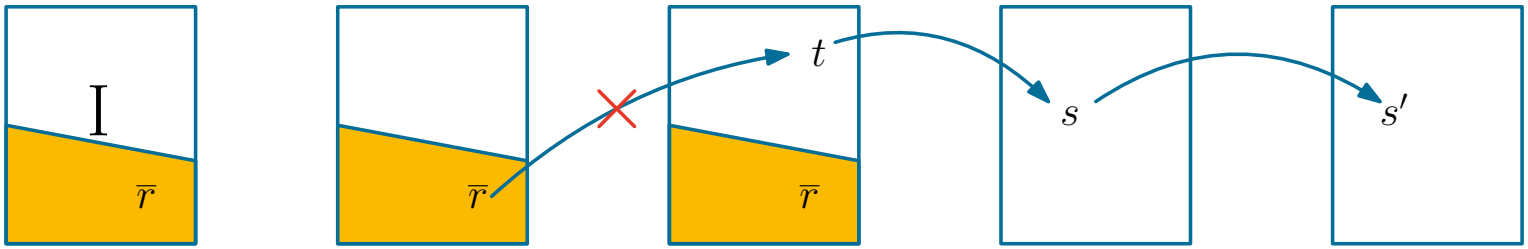
IC3



$$\underbrace{\overline{s}}_{\text{Constraint}} \wedge \underbrace{Frame_{i-1} \wedge T}_{\text{Clauses}} \wedge \underbrace{s'}_{\text{Assumptions}}$$



IC3



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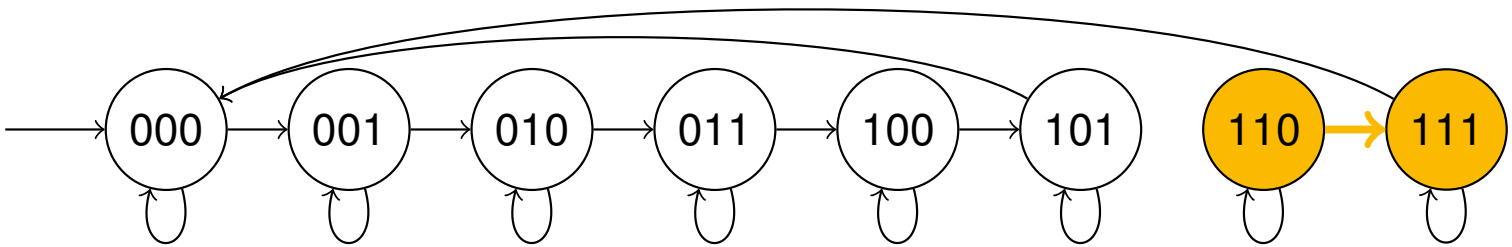
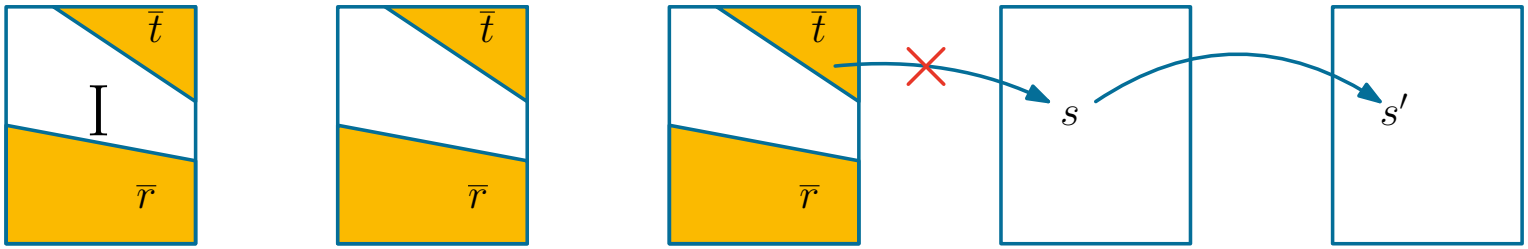


IC3



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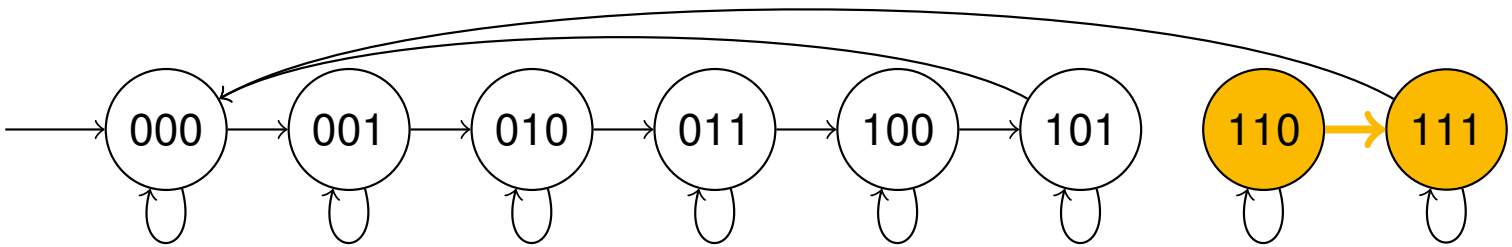
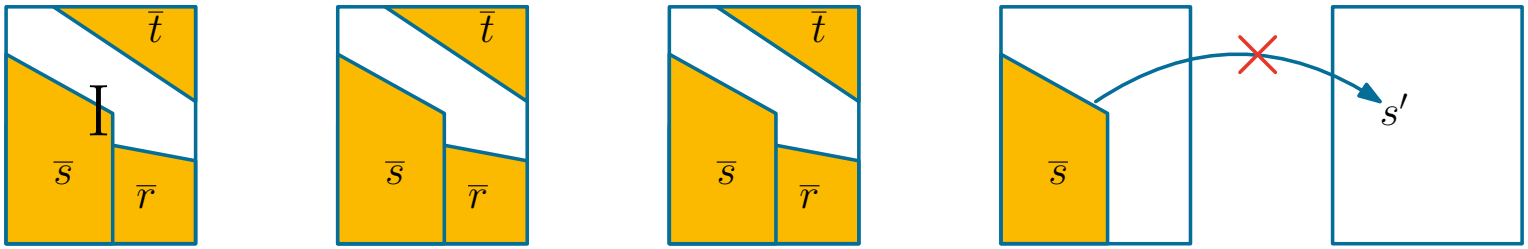
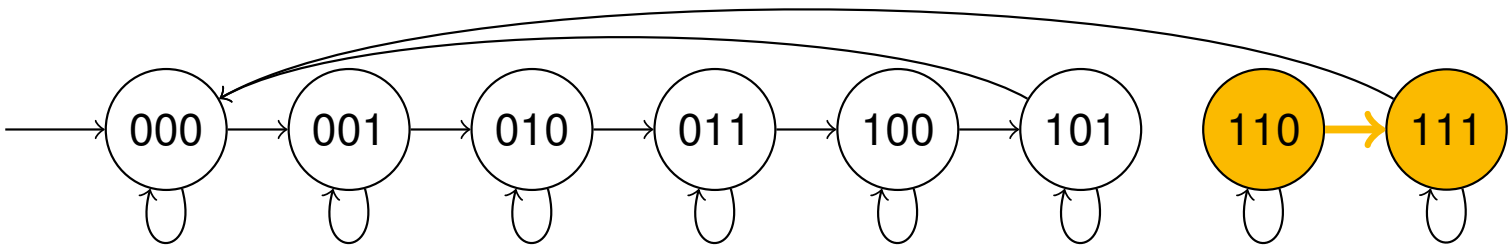


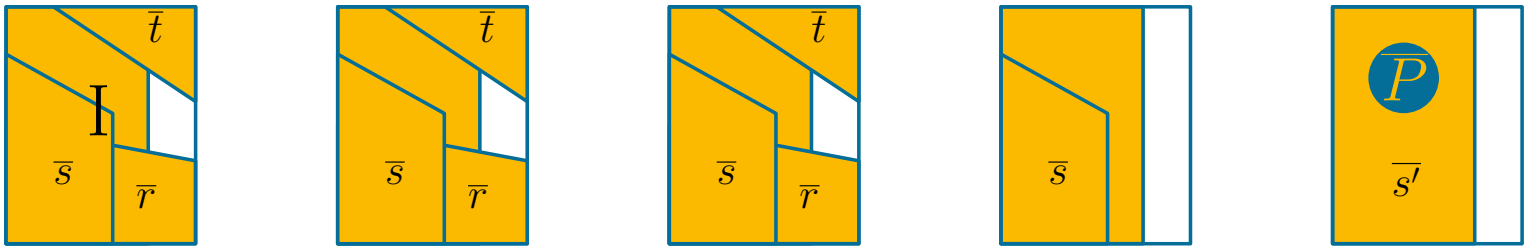
IC3



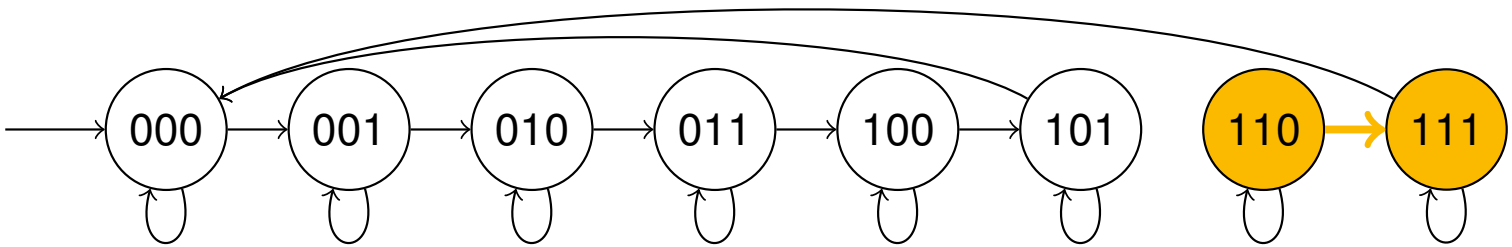
$$\underbrace{\overline{s}}_{\text{Constraint}} \wedge \underbrace{Frame_{i-1} \wedge T}_{\text{Clauses}} \wedge \underbrace{s'}_{\text{Assumptions}}$$



IC3



$$\underbrace{\overline{s}}_{\text{Constraint}} \wedge \underbrace{Frame_{i-1} \wedge T}_{\text{Clauses}} \wedge \underbrace{s'}_{\text{Assumptions}}$$



CaDiCaL

> *IC3 is the first widespread verification method that requires highly efficient incremental solvers. An incremental interface for IC3 must allow single clauses to be pushed and popped; it must also allow literal assumptions. [It poses] many thousands to millions of queries in the course of an analysis, and so speed is crucial.*
>
> *Understanding IC3 [Bradley, Aaron R., 2012]*

github.com/arminbiere/cadical

Model-based testing: 1.5 Billion runs

Activation Literal

$$\underbrace{\overline{s}}_{\text{Constraint}} \wedge \underbrace{Frame_{i-1} \wedge T}_{\text{Clauses}} \wedge \underbrace{s'}_{\text{Assumptions}}$$

$$\underbrace{(a \vee \overline{s}) \wedge Frame_{i-1} \wedge T}_{\text{Clauses}} \wedge \underbrace{s' \wedge \overline{a}}_{\text{Assumptions}}$$

- Deactivated Clauses
- Unused activation literals
- Solver restarts – SAT solver management strategies in IC3

  [Cabodi, G. and Camurati, P. E. and Mishchenko, A. and Palena, M. and Pasini, P., 2017]
- Worse Learned Clauses
  - Deleted by restarts
  - Tend to contain the activation literal

$\Rightarrow$ More failed assumptions

$\Rightarrow$ Worse generalization

Experimental Setup

- Bit-level model checker ABC
  - Multi-engine
  - Sophisticated preprocessing
- Replaced MiniSat [Eén, Niklas and Sörensson, Niklas, 2004]
  with CaDiCaL
  [Biere, Armin and Fleury, Mathias and Heisinger, Maximilian and Fazekas, Katalin, 2020]
- Replaced activation literals with constraints
- Benchmarks HWMCC'19 [Preiner, Mathias and Biere, Armin, 2019]
  - 219 instances, 15 unsolved and removed

PAR-2 Score

	Di	Og	Ca	Co	De
Mean	80	46	16	8.93	**8.21**
beemTele6Int	136	7200	**53**	181	101
toyLock4	7200	483	1731	**357**	359
visArraysField5	7200	1.6	**0.58**	51	34
nan	208	421	163	158	**140**
beemColl6Int	241	258	322	133	**108**
cal110	213	168	130	**110**	122
cal109	179	197	102	117	**86**
cal93	186	136	121	**118**	140
cal94	127	160	115	**95**	131
cal100	112	**42**	67	67	54

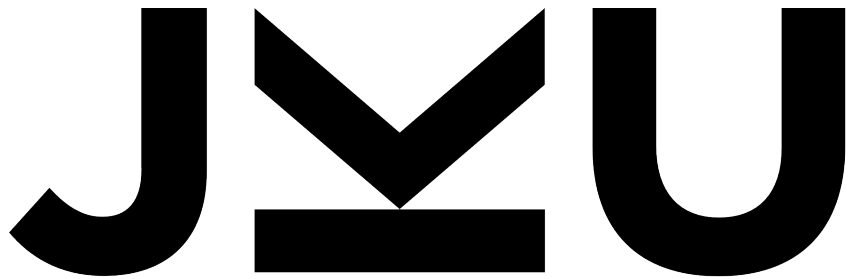
Disable restarts, **O**ri**g**inal ABC, **Ca**DiCaL,
Constraints, **De**fer model reconstruction

Time Per Call

	Restarts	Calls [10^3]		TpC [ms]	
	Ca	Ca	Co	Ca	Co
Mean	61	19	**15**	0.61	**0.51**
beemTele6Int	520	**157**	574	**0.24**	0.27
toyLock4	7459	2251	**1098**	0.42	**0.25**
visArraysField5	1	**<1**	113	0.53	**0.41**
nan	1381	**420**	423	**0.29**	0.32
beemColl6Int	398	123	**91**	2.31	**1.24**
cal110	191	59	**42**	**1.96**	2.39
cal109	110	**34**	44	2.71	**2.44**
cal93	206	63	**58**	**1.69**	1.8
cal94	171	52	**41**	**1.94**	2.1
cal100	148	45	**44**	**1.23**	1.29

Conclusion

- Our approach
  - Easy to implement in modern SAT solvers
  - Simplifies solver usage
  - Speedup of 1.84 for IC3
- Future work
  - Multiple constraints for other applications
  - Reduce number of failed literals



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