Verification-Aided Deep Ensemble Selection

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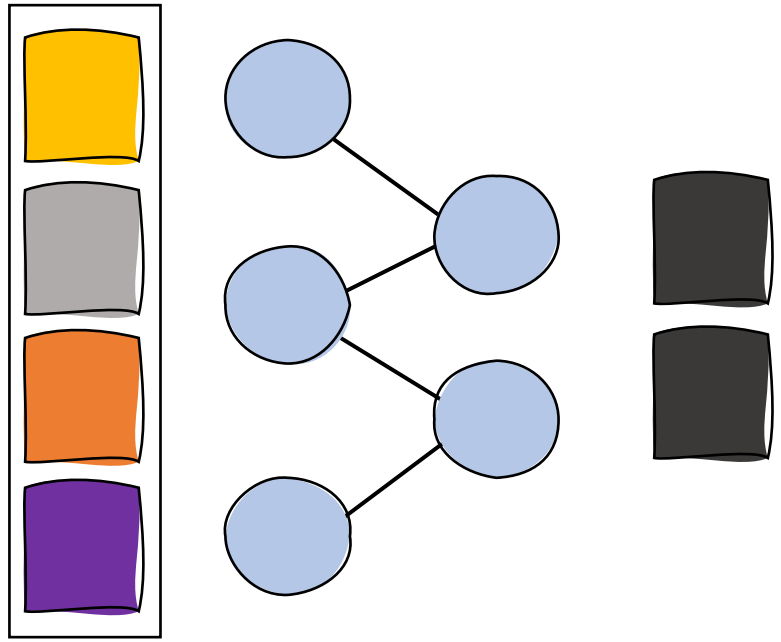
Guy Amir Tom Zelazny
Guy Katz Michael Schapira

fmcad

Motivation

Deep Neural Networks (DNNs) achieve state-of-the-art results, but:



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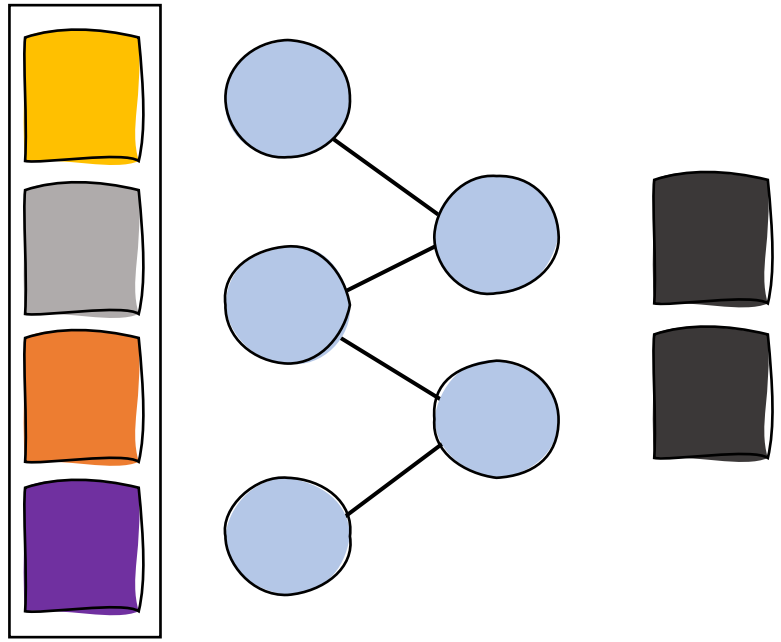

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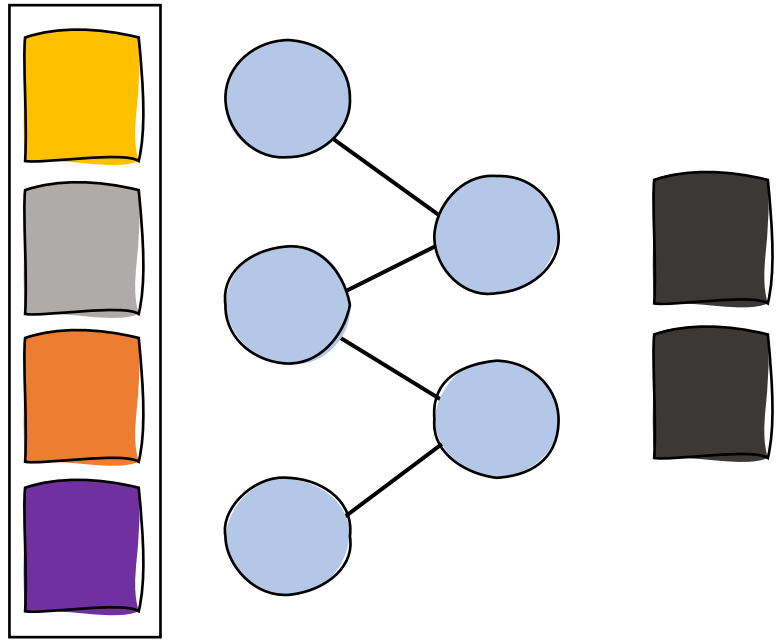

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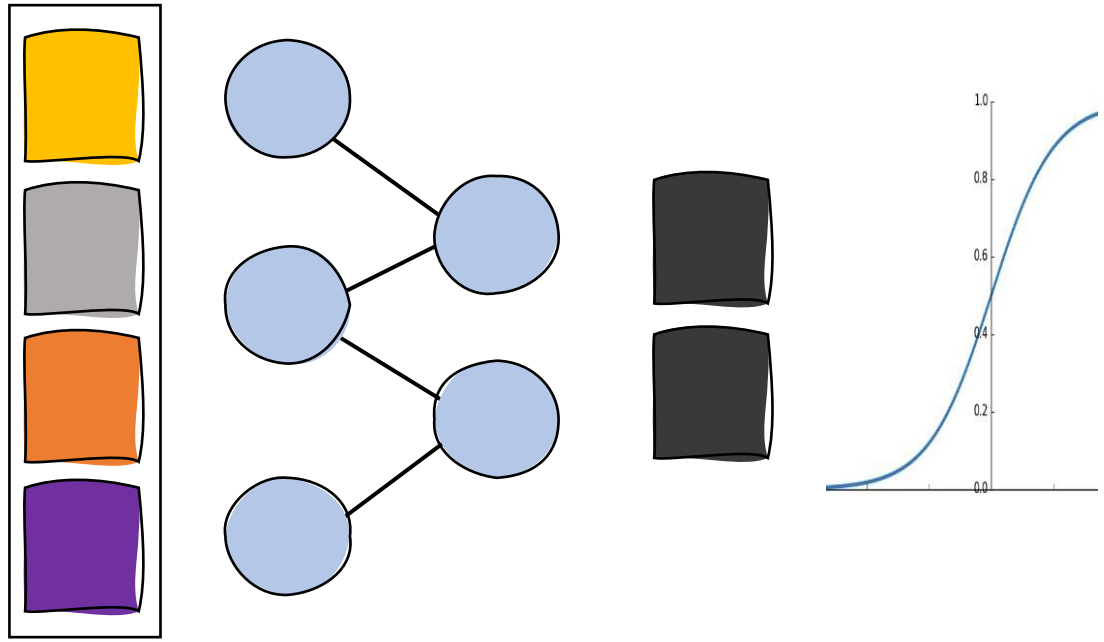
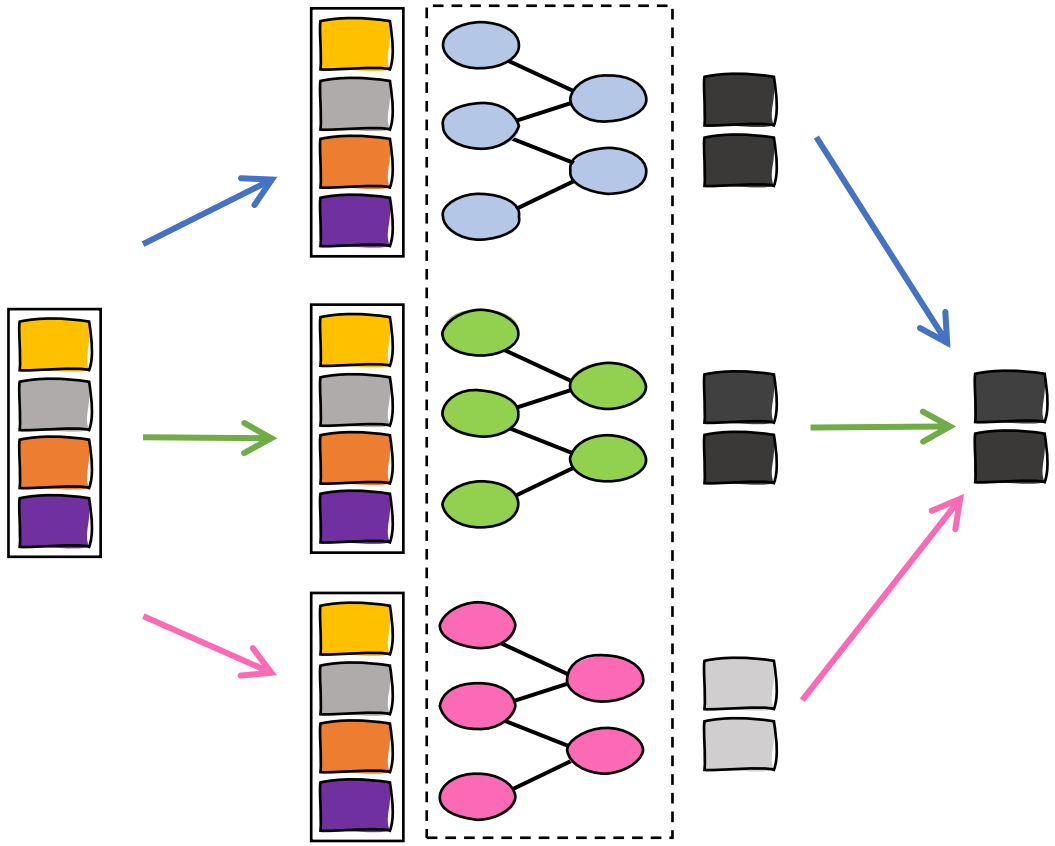


Solution?

Train **ensembles** of DNN models



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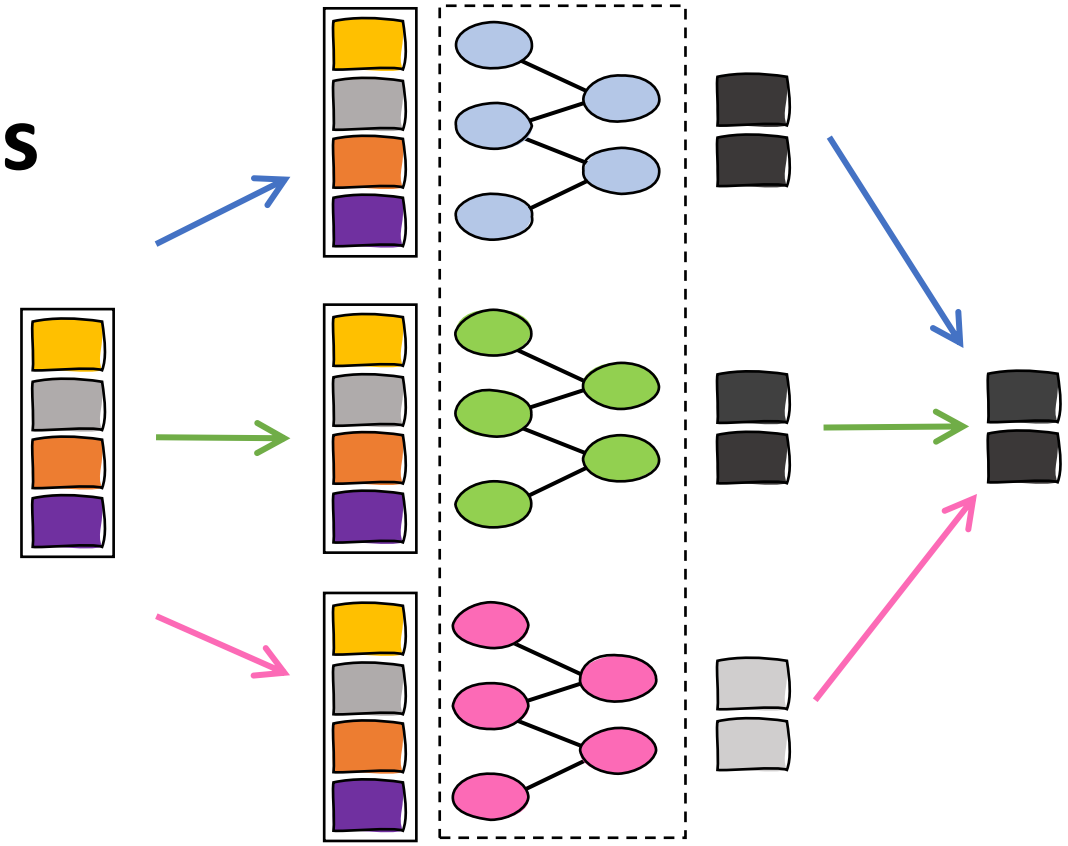

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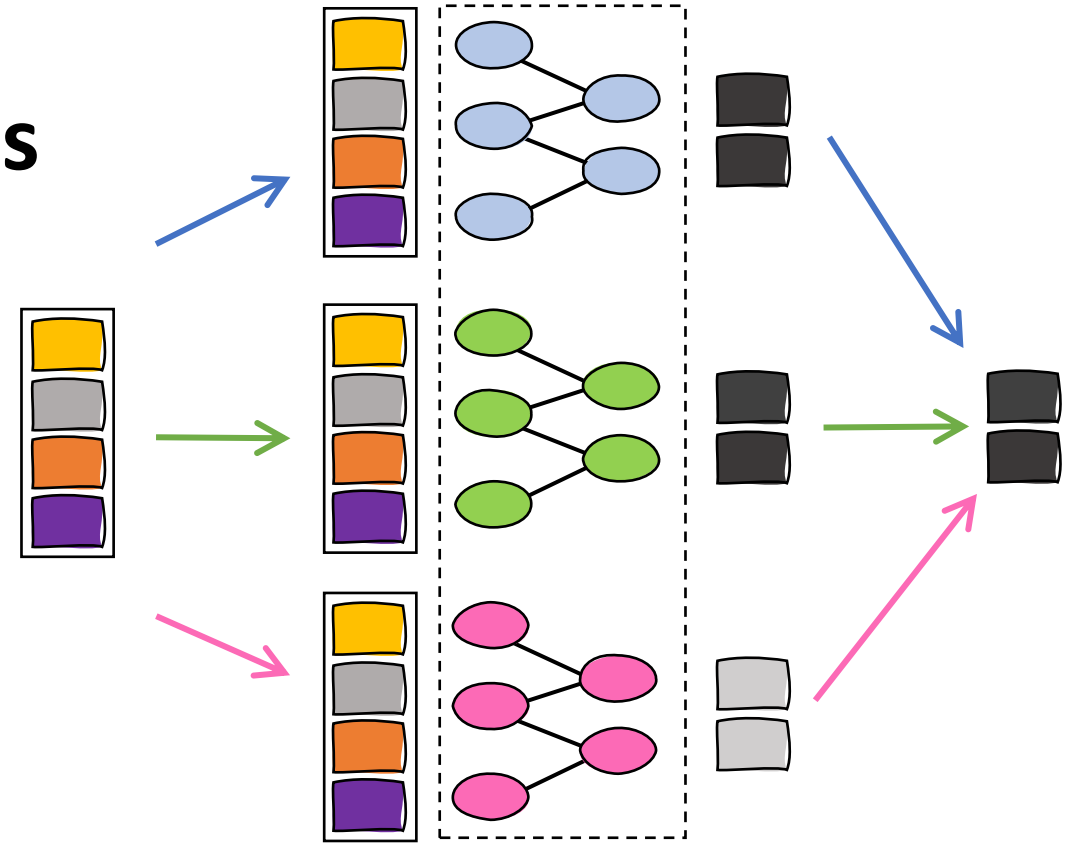
Train **ensembles** of DNN models

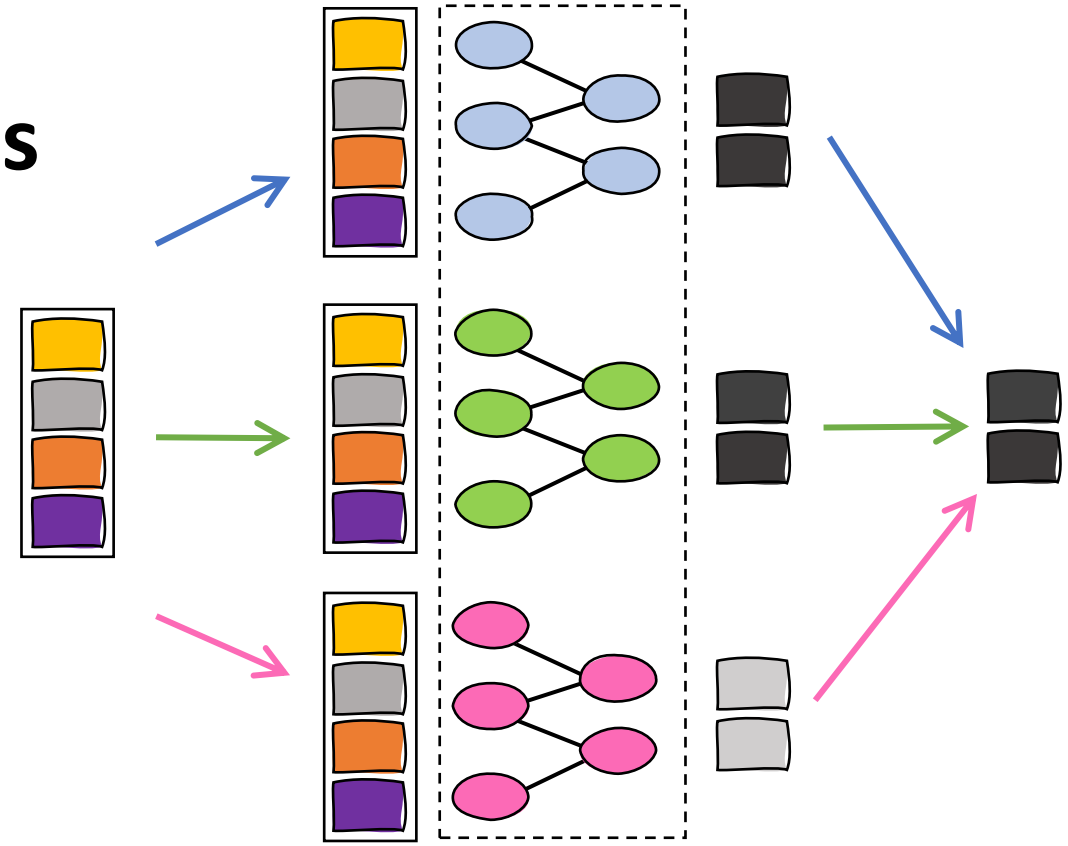
- ☑ Higher **accuracy** and **robustness**
- ☑ **Uncertainty** estimation



Solution?

Train **ensembles** of DNN models

- Higher **accuracy** and **robustness**
- **Uncertainty** estimation
- **Parallelize** training



Ensemble Selection

Ensemble Pruning - Given $n > k$ trained DNNs, how can we choose the **``best''** k-sized ensemble ?

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most diverse

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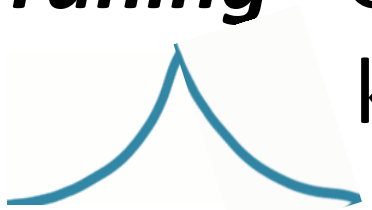
Ensemble Pruning - Given $n > k$ trained DNNs, how can we choose the  k-sized ensemble ?

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✗ **"Ensembling Neural Networks: Many Could be Better Than All"**
Zhou et. al. (2010)

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use **formal verification** to select the best subset k

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✗ Each query is **NP-hard**

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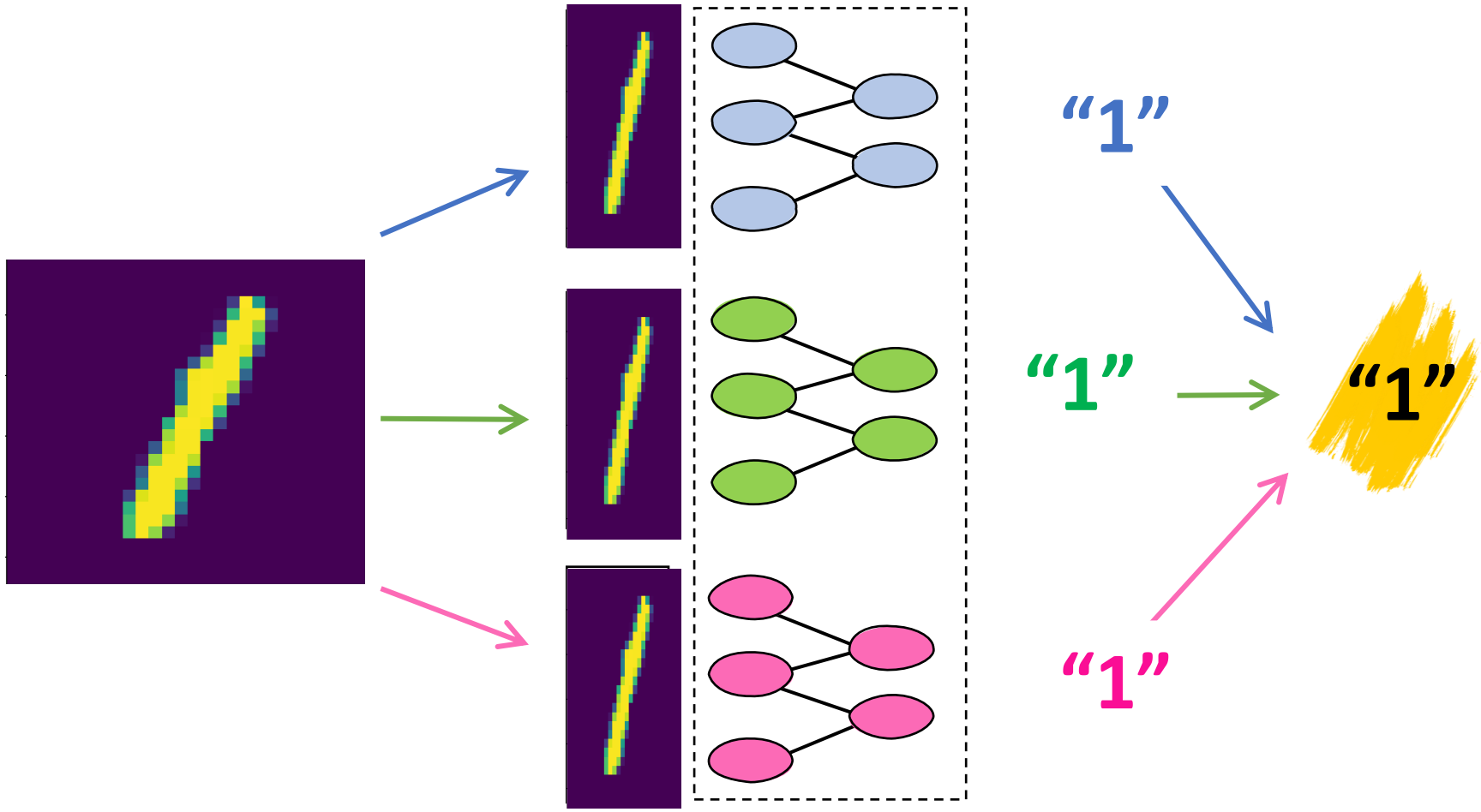
Check if DNNs tend to err simultaneously

Agreement Points



Agreement Points

Data points from the test set which are ***classified the same*** by all DNN members

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Intuitively – areas with a ***high consensus*** among the DNNs

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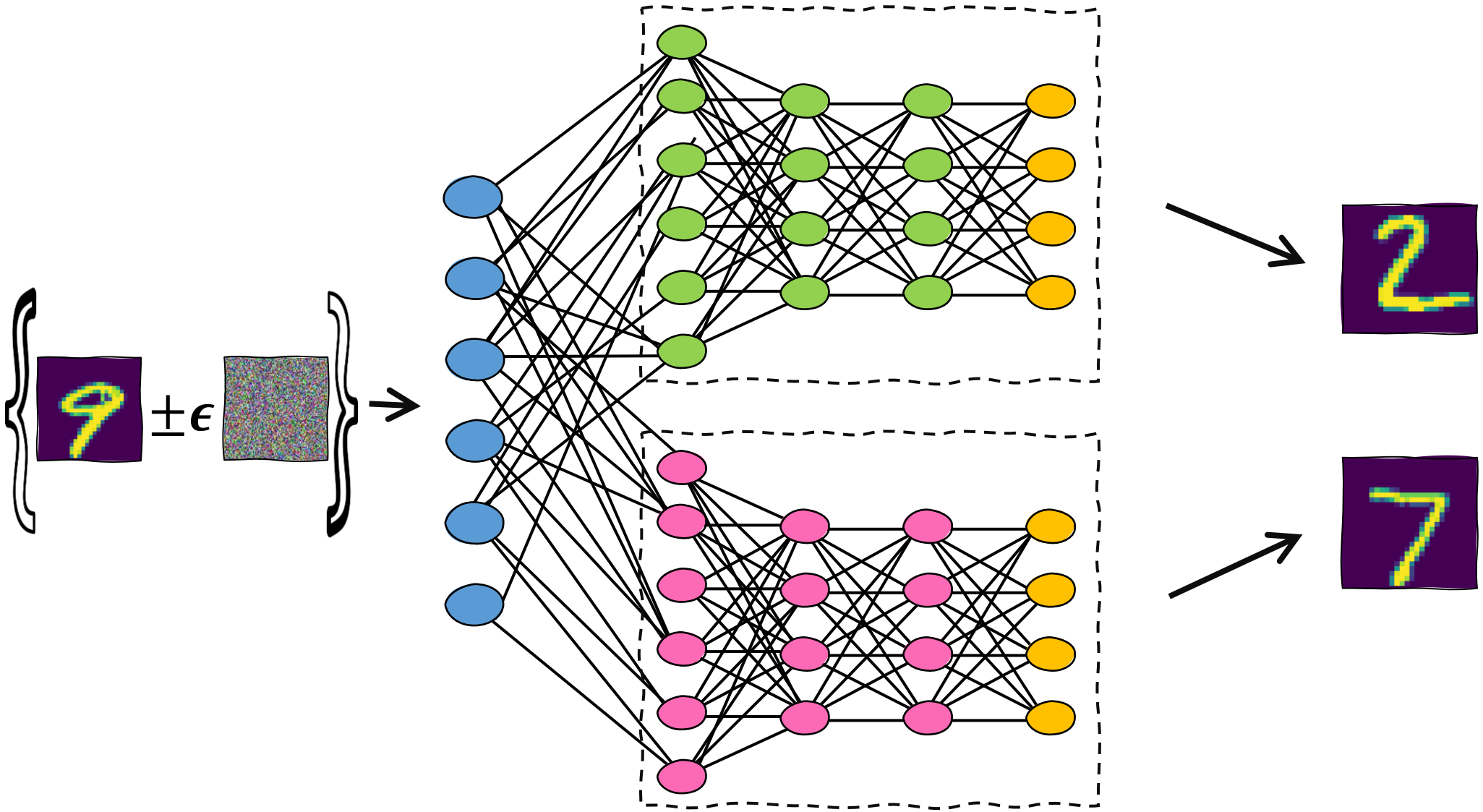
Allows a ***fair comparison*** between DNN pairs

Mutual Error

Given a fixed ***agreement point*** and a ***pair*** of DNNs

A ***mutual error*** is a perturbation that ***simultaneously*** causes a pair of DNNs to ***misclassify*** the point

Mutual Error (with verifier)



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The **higher** the ***uniqueness score*** - the **better** the DNN

Greedy-Based Strategy

Initially:

1 Independently ***train n DNNs*** and fix a set of ***agreement points***

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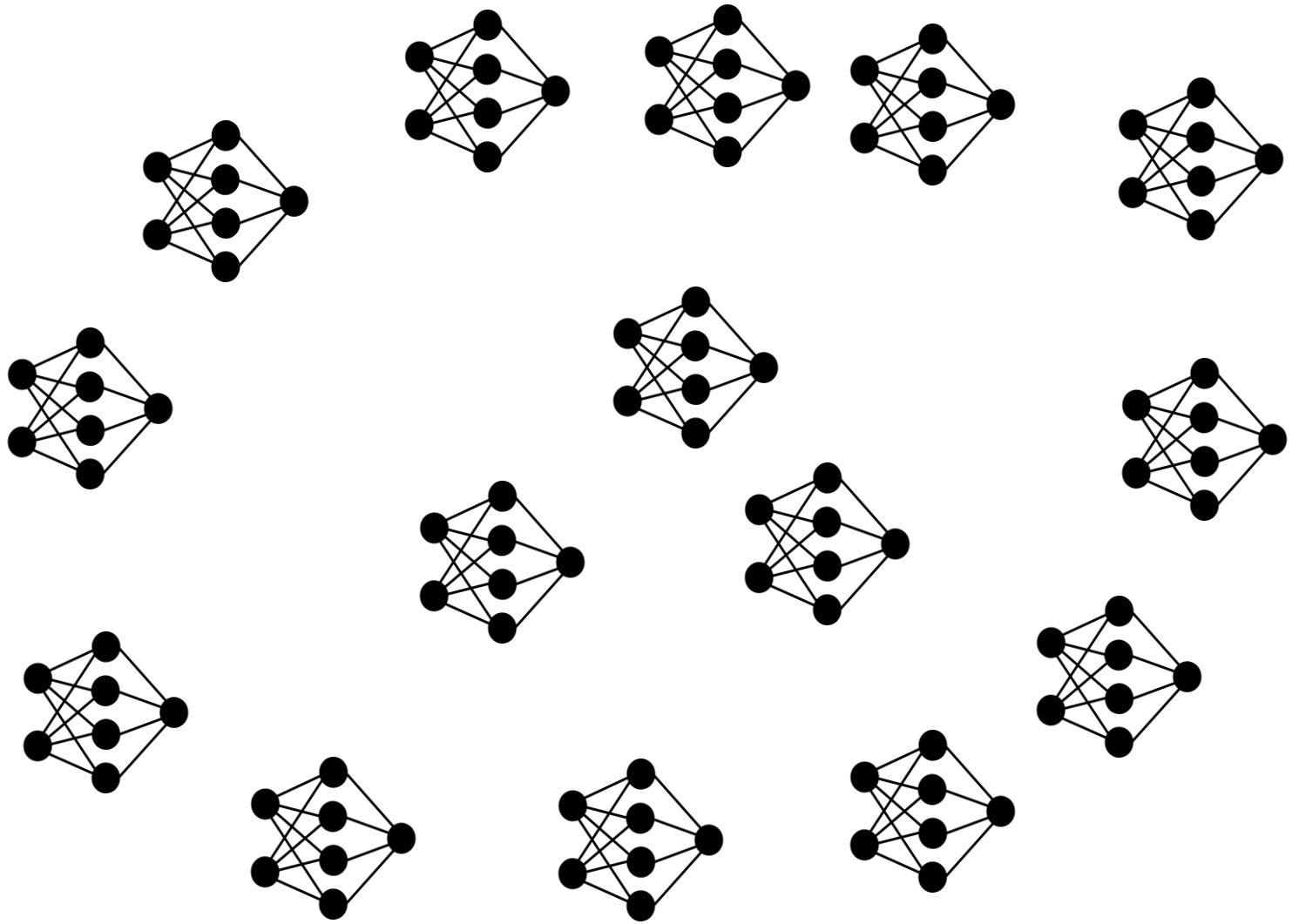

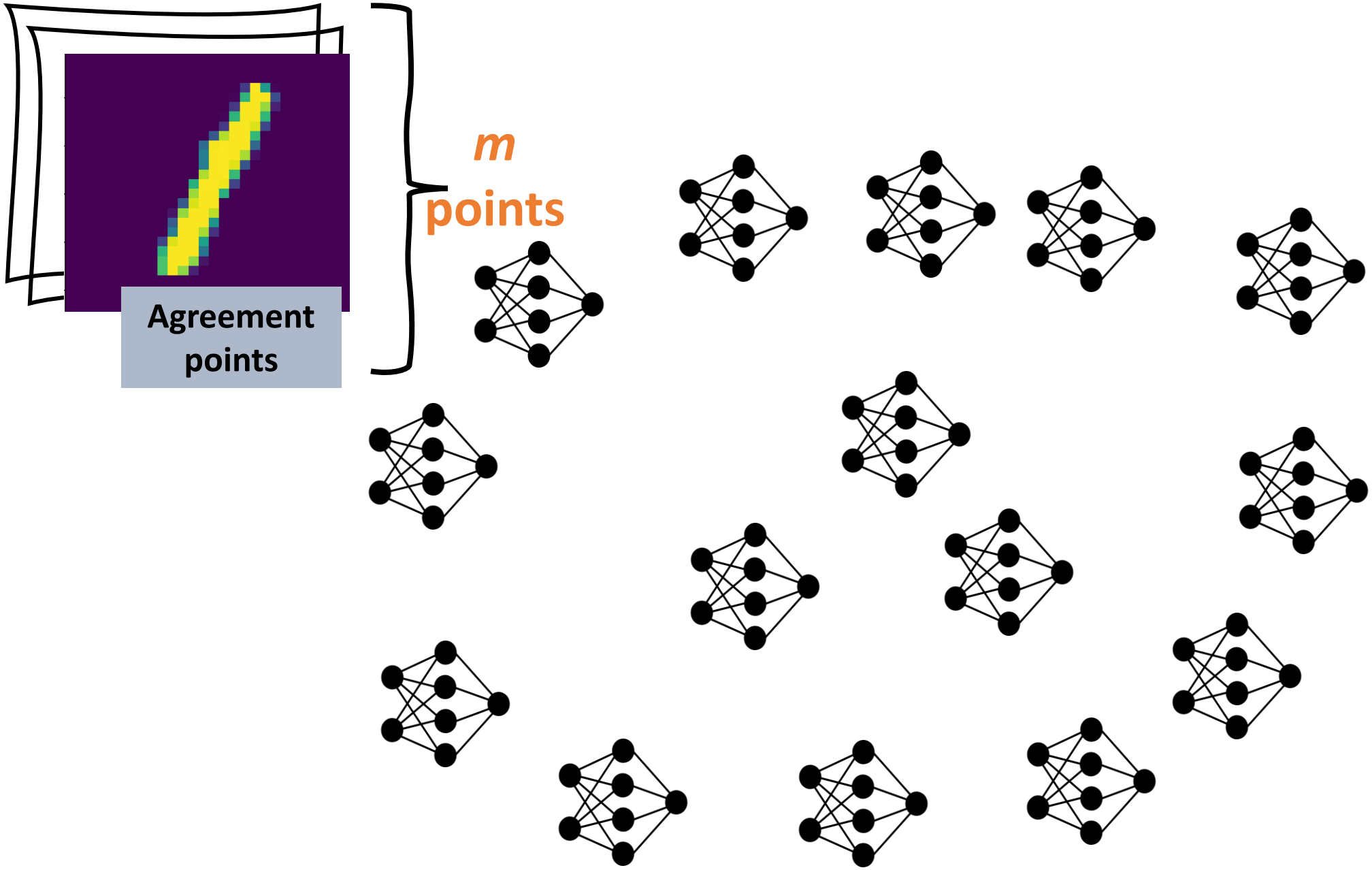
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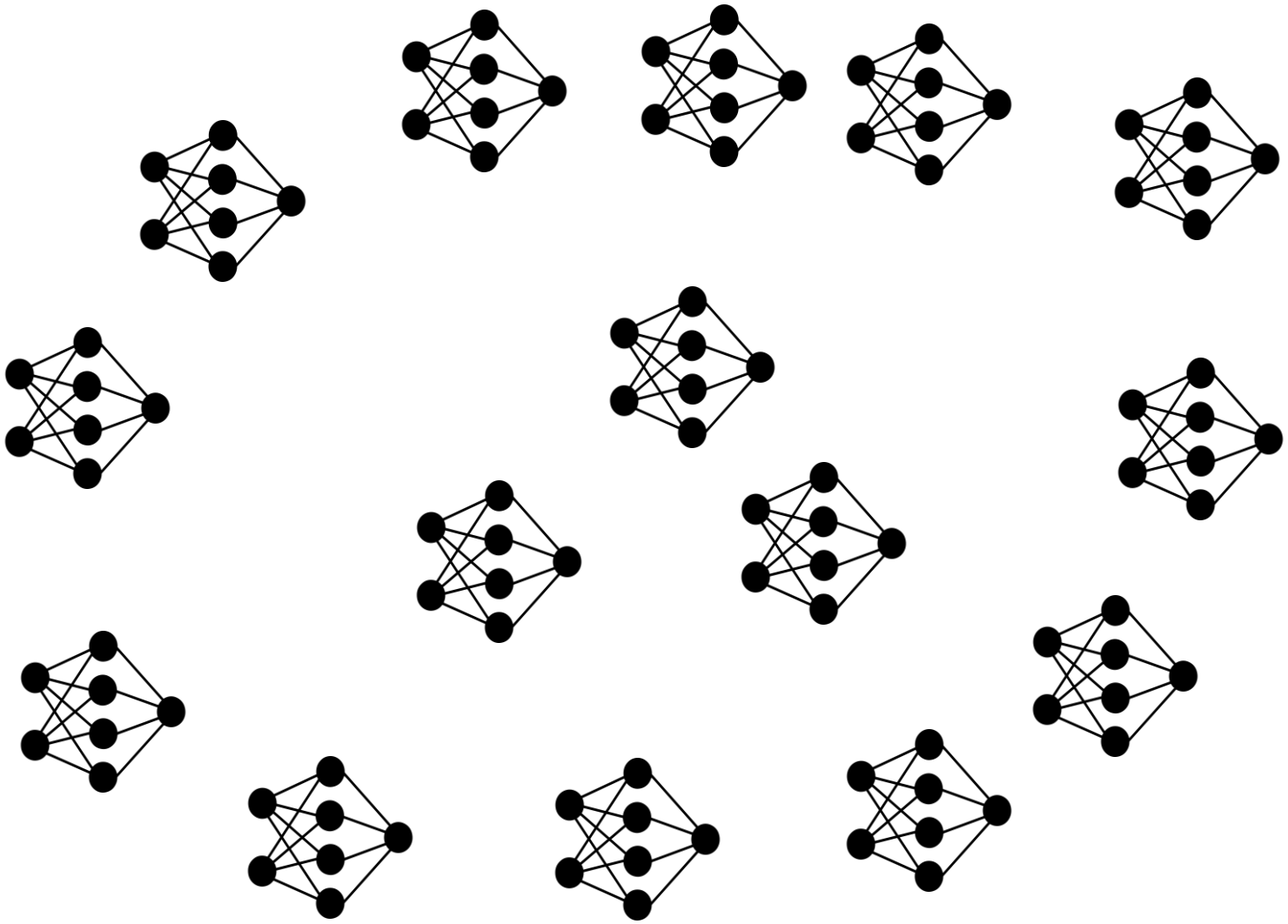
points
Agreement
points

$$k = 3 < 15 = n$$



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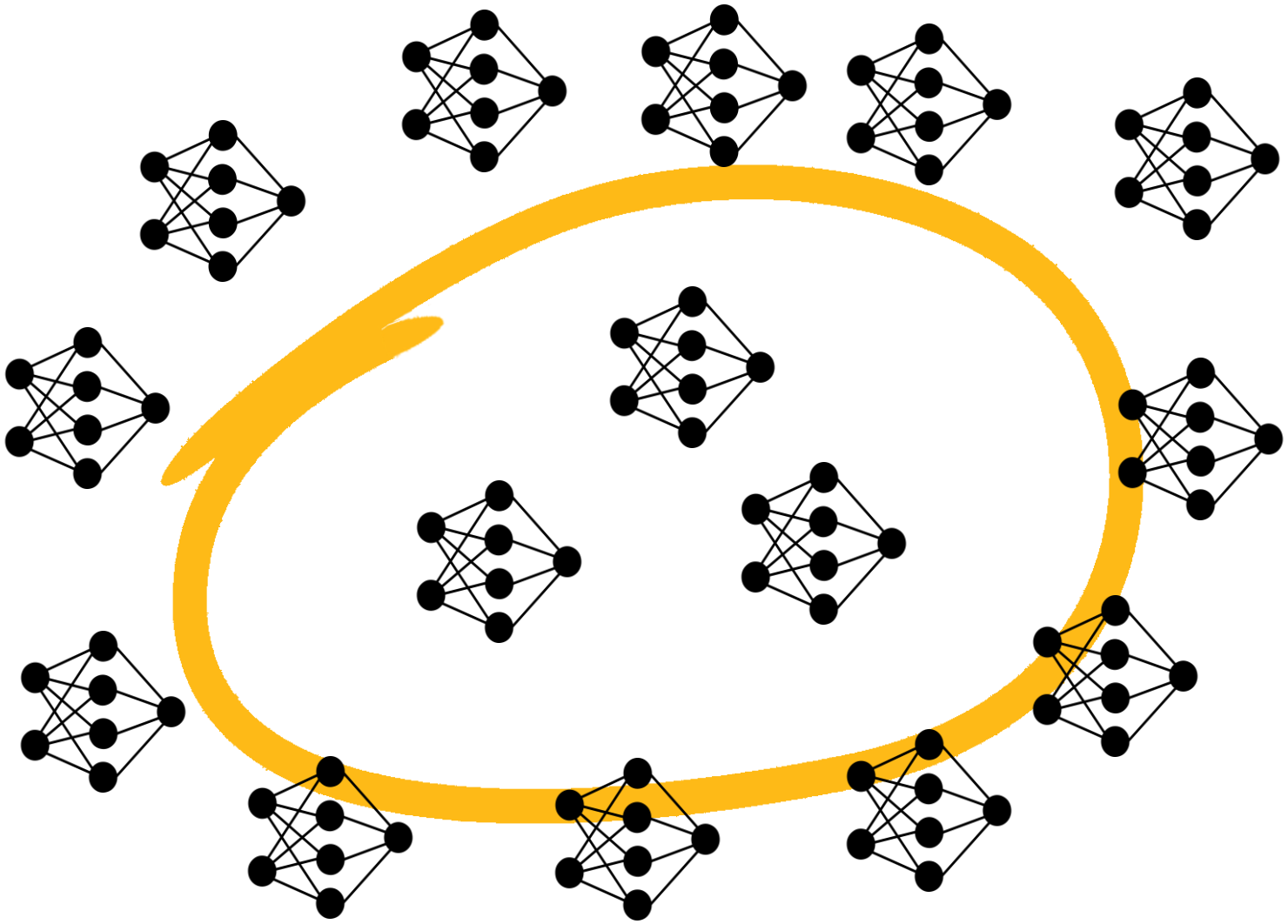

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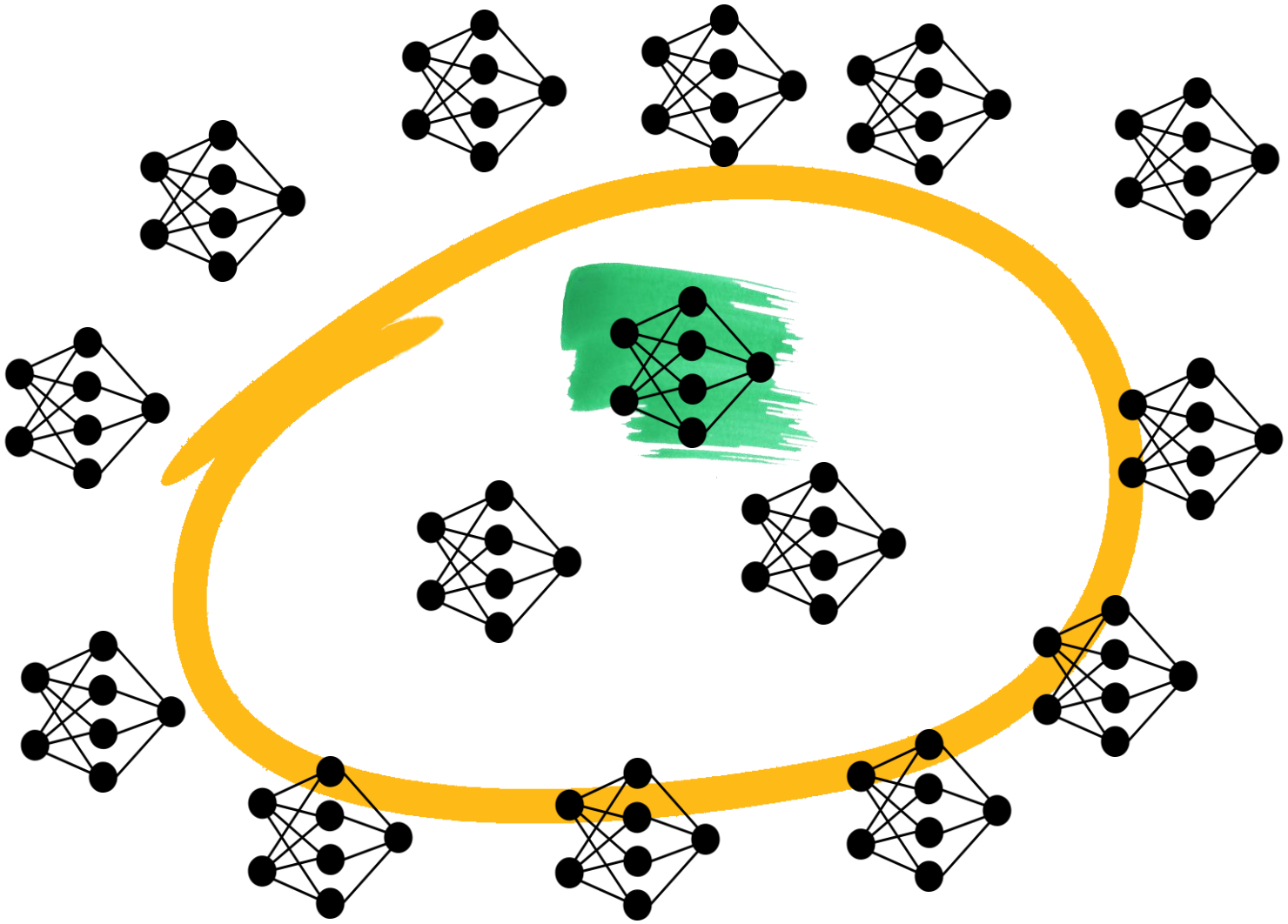

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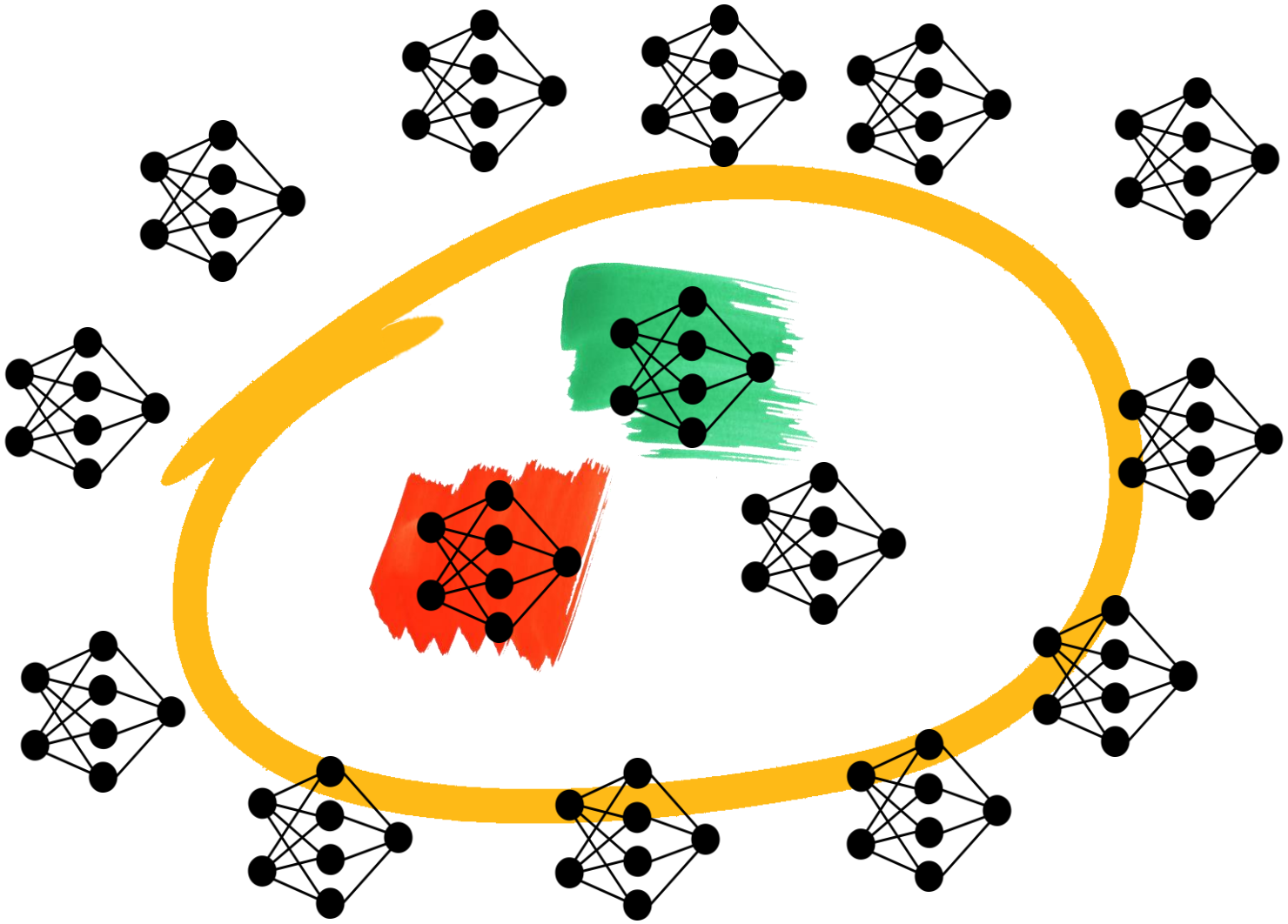

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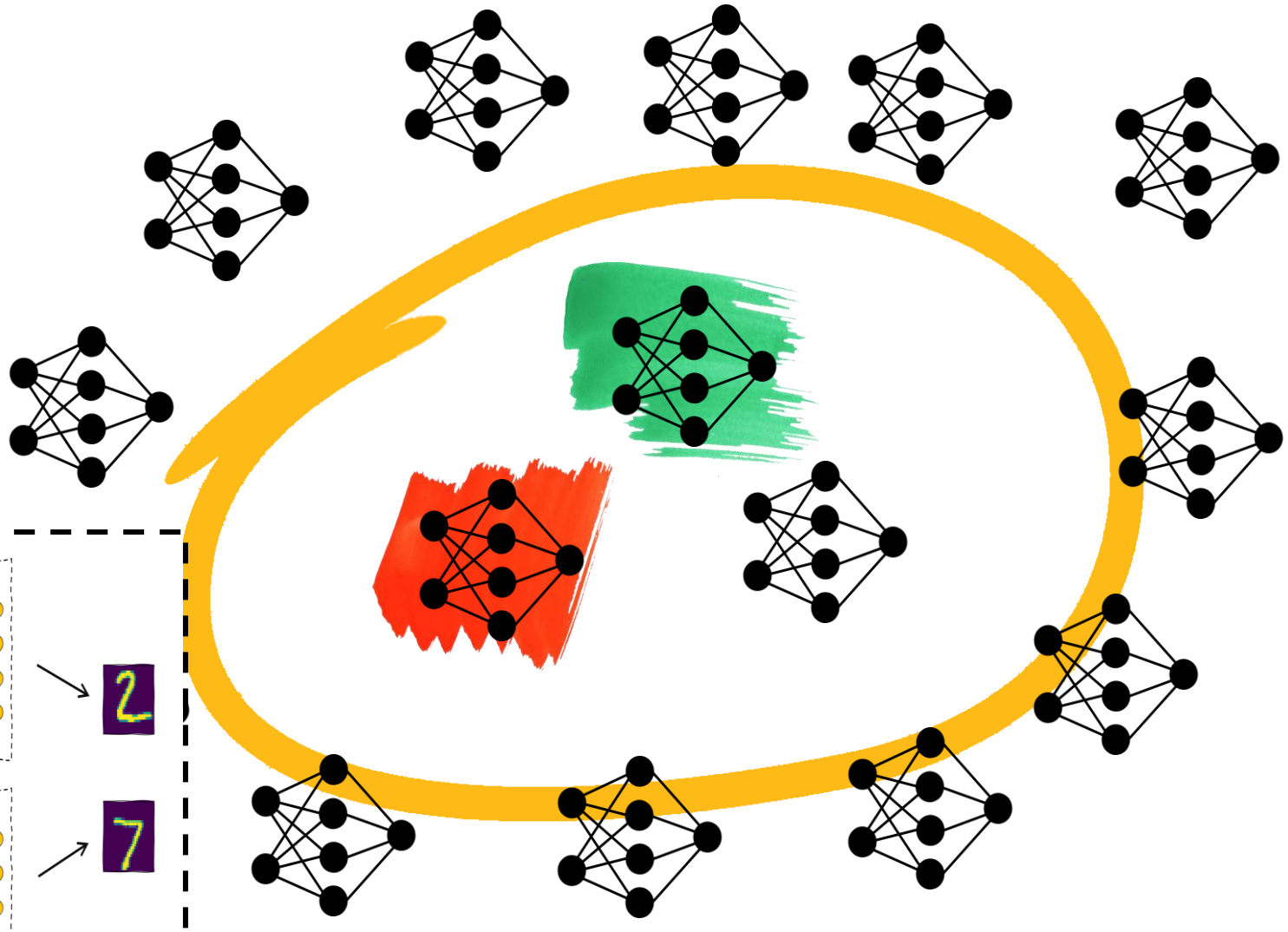

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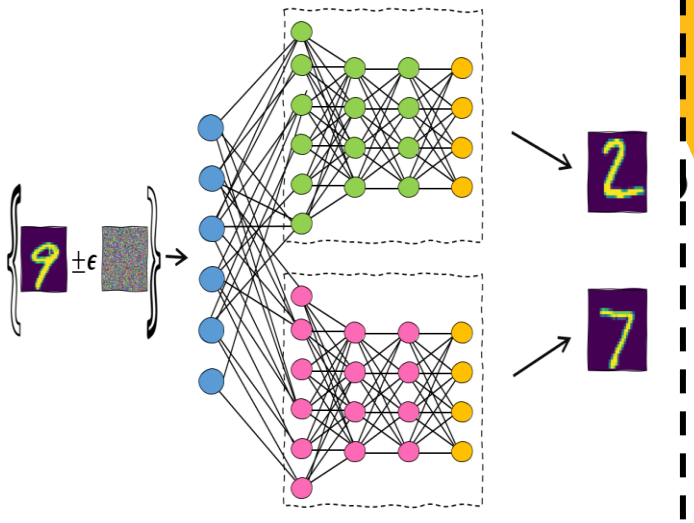

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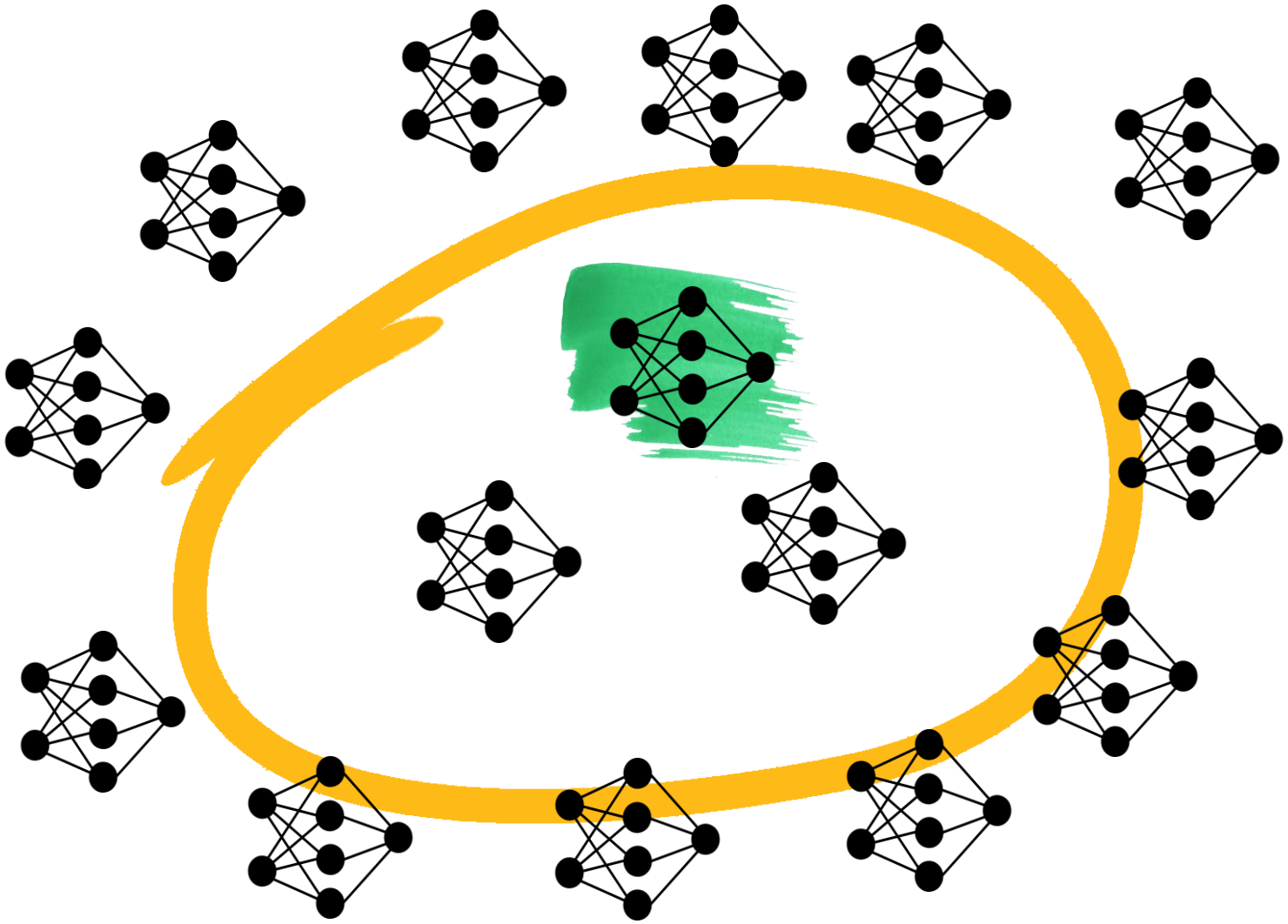

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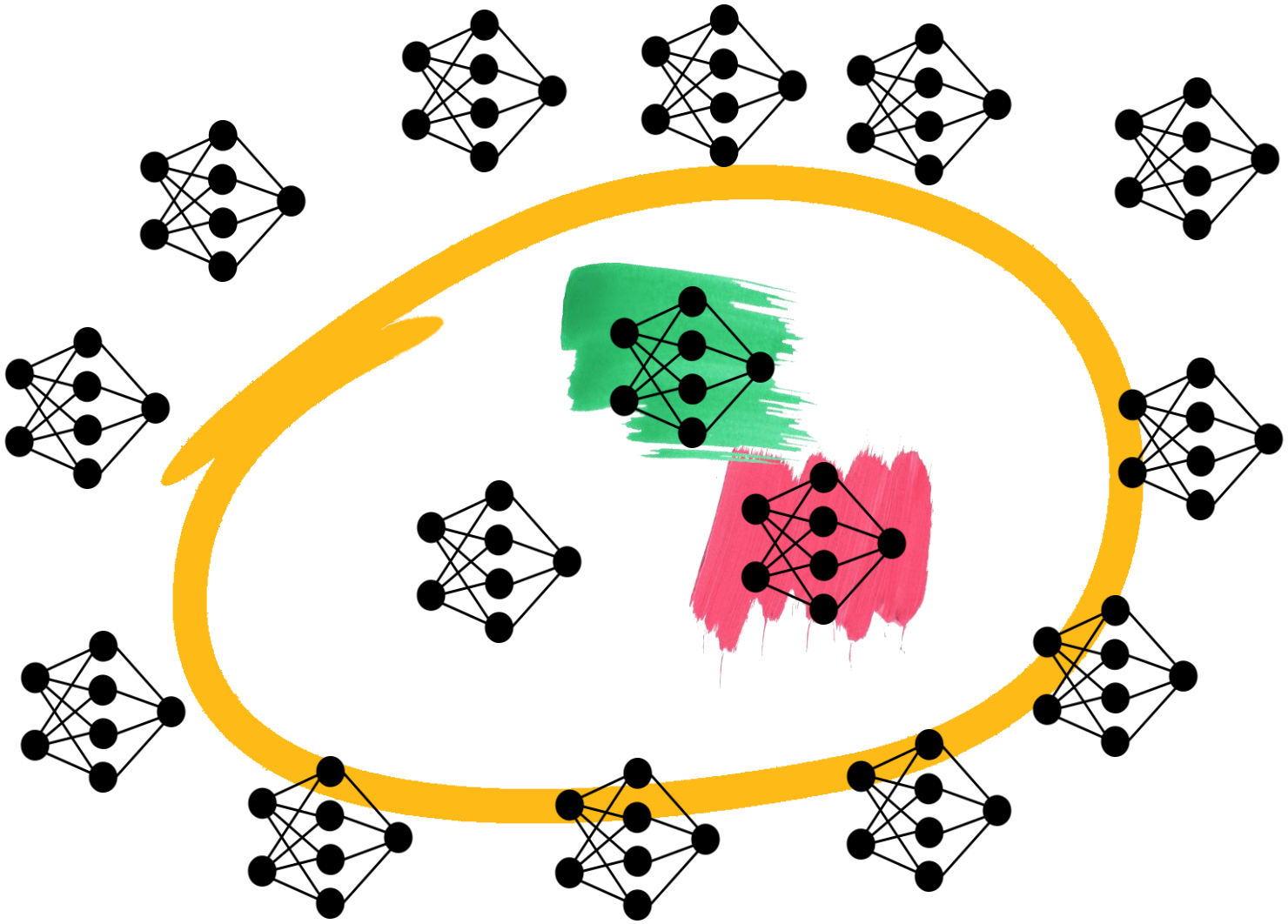

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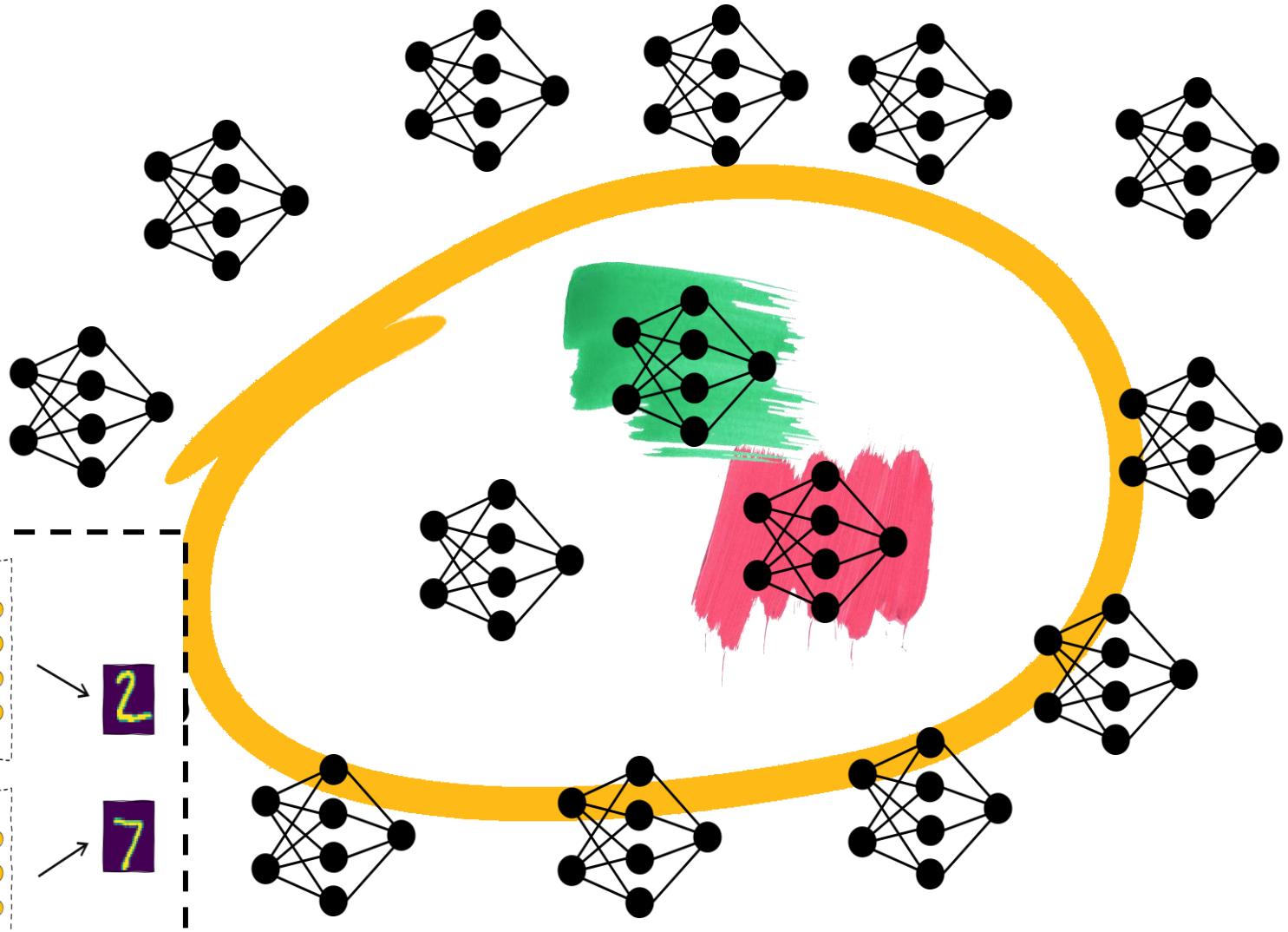

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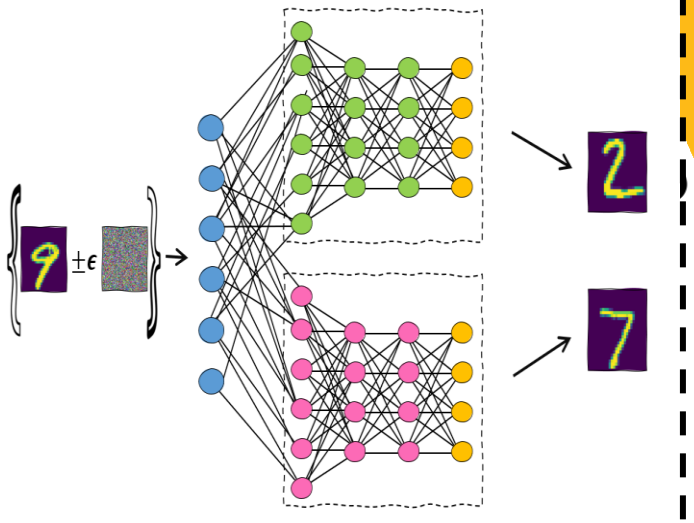

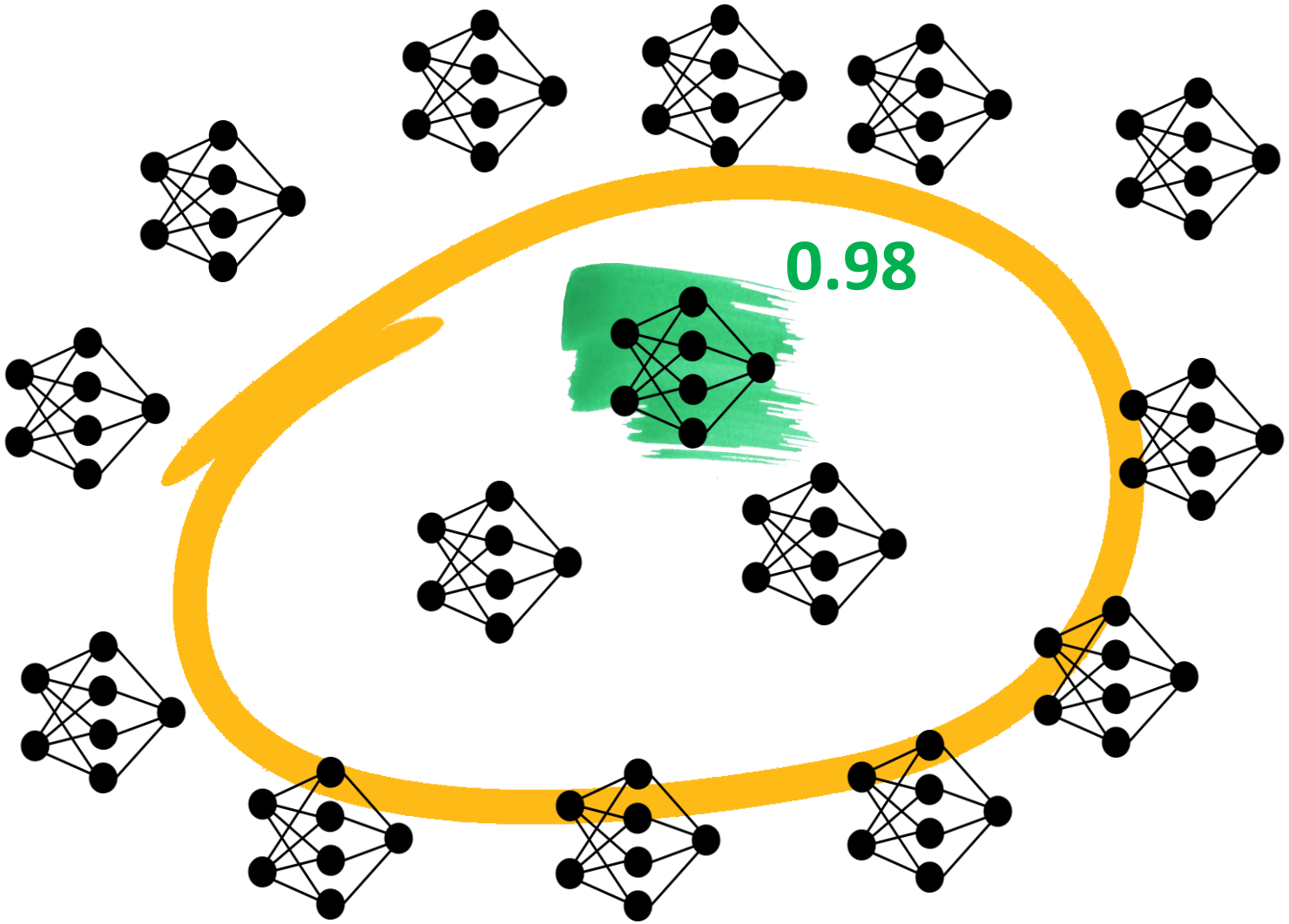
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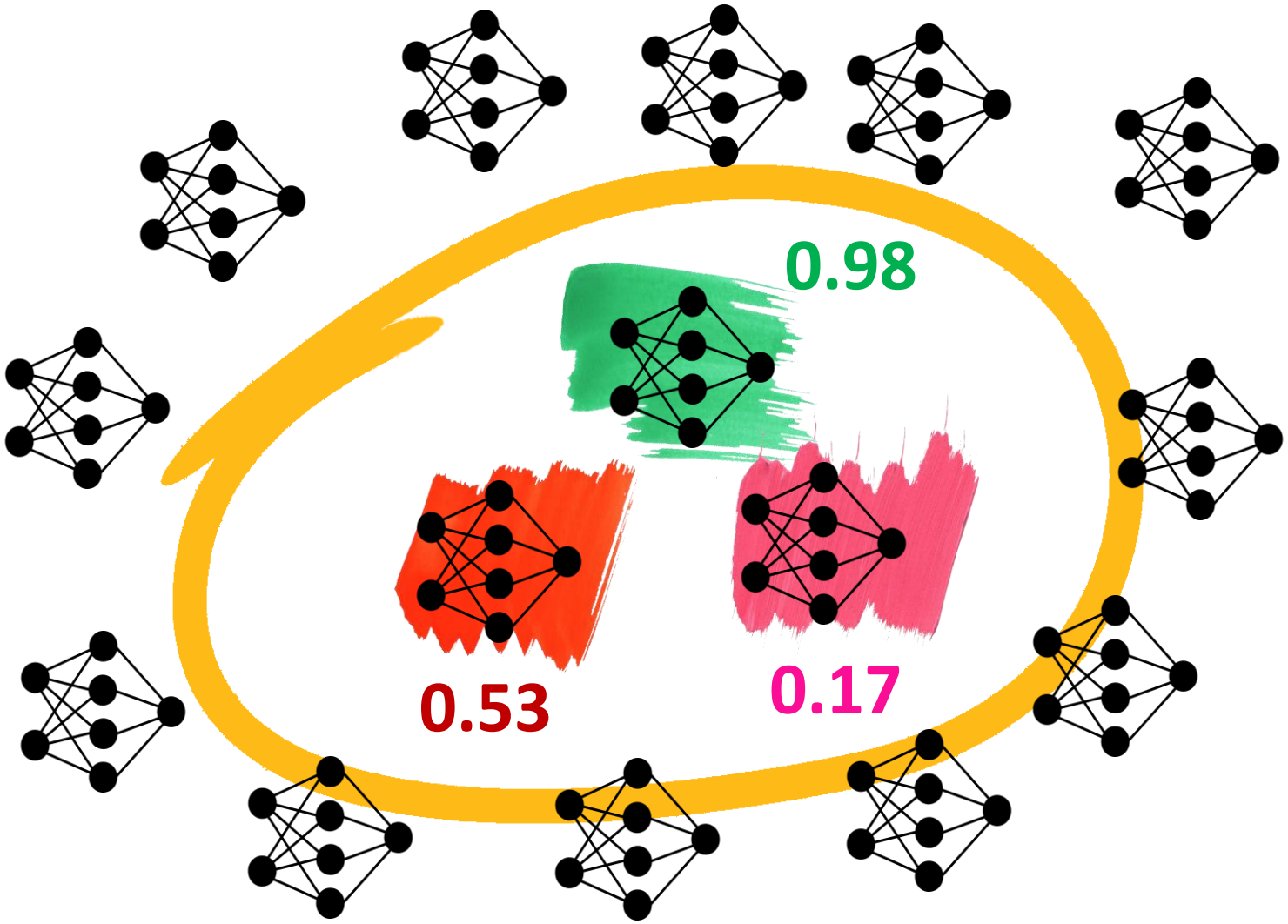
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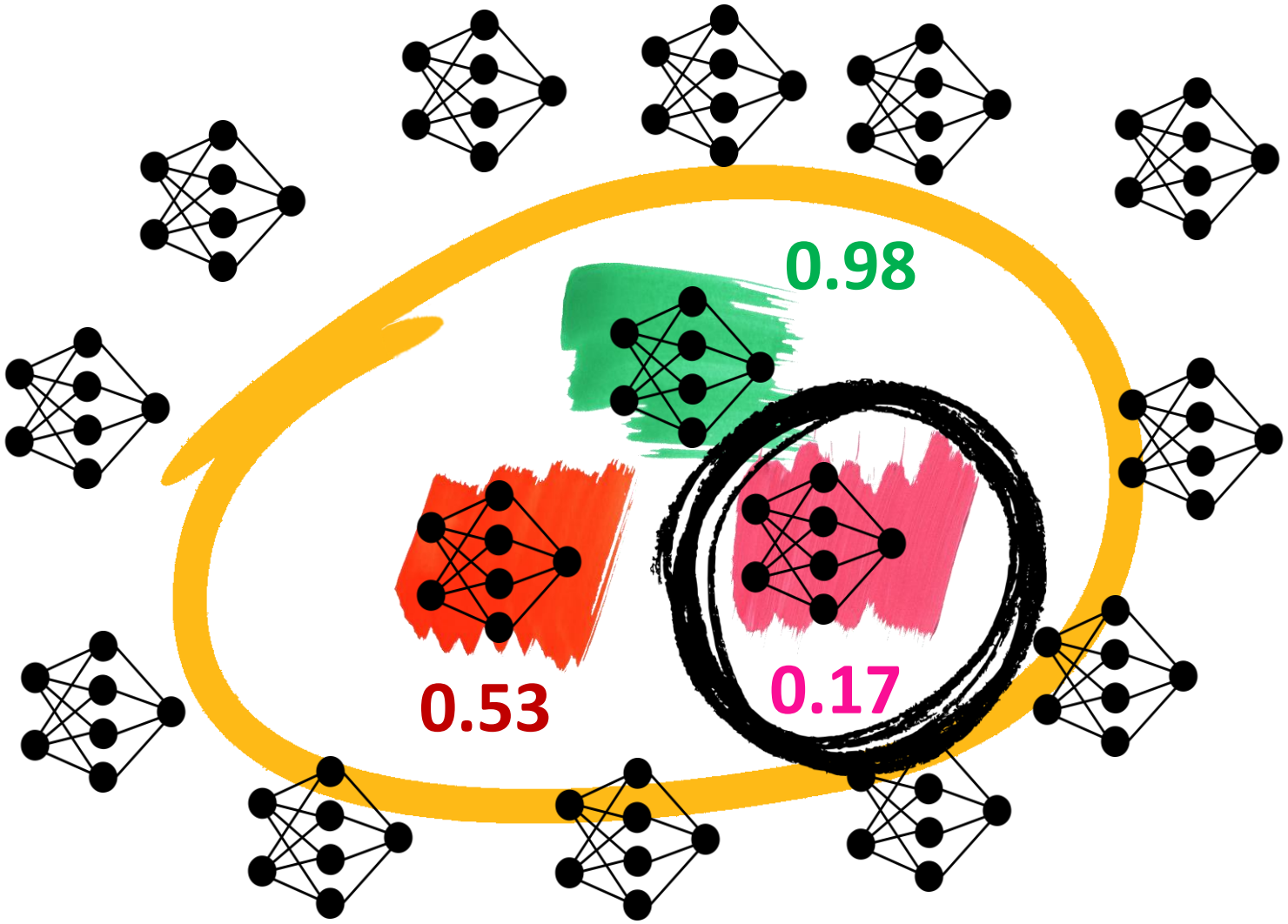
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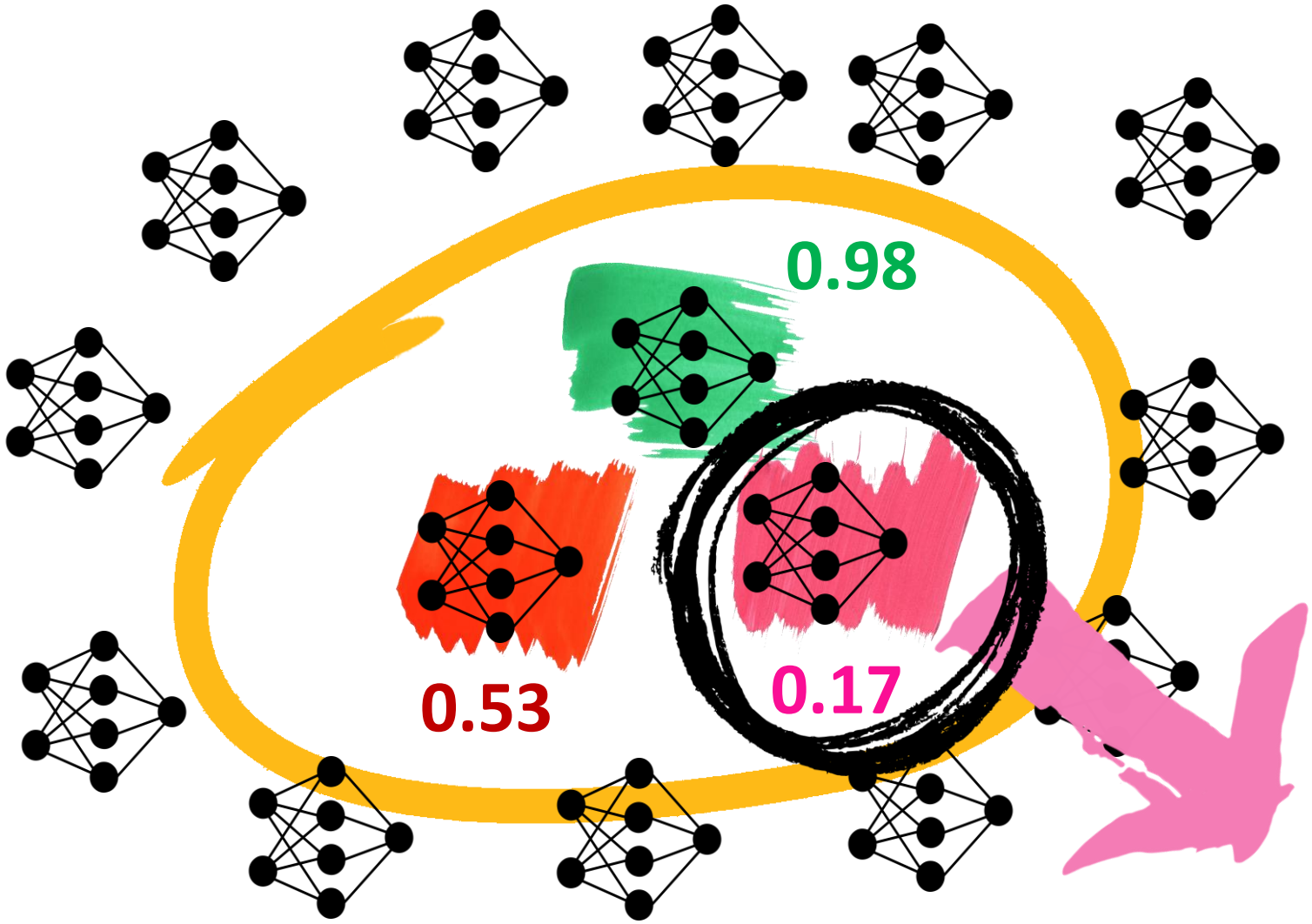
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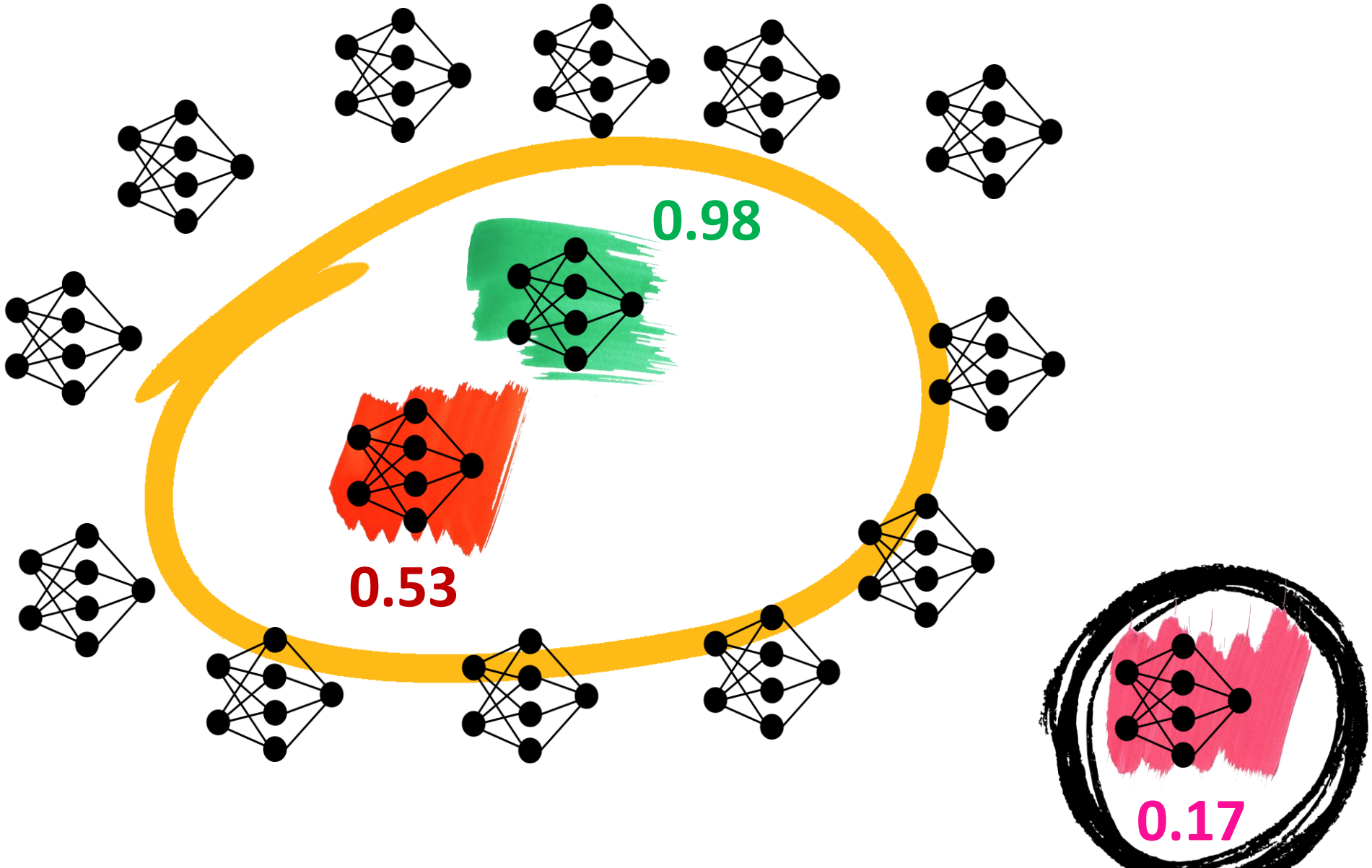
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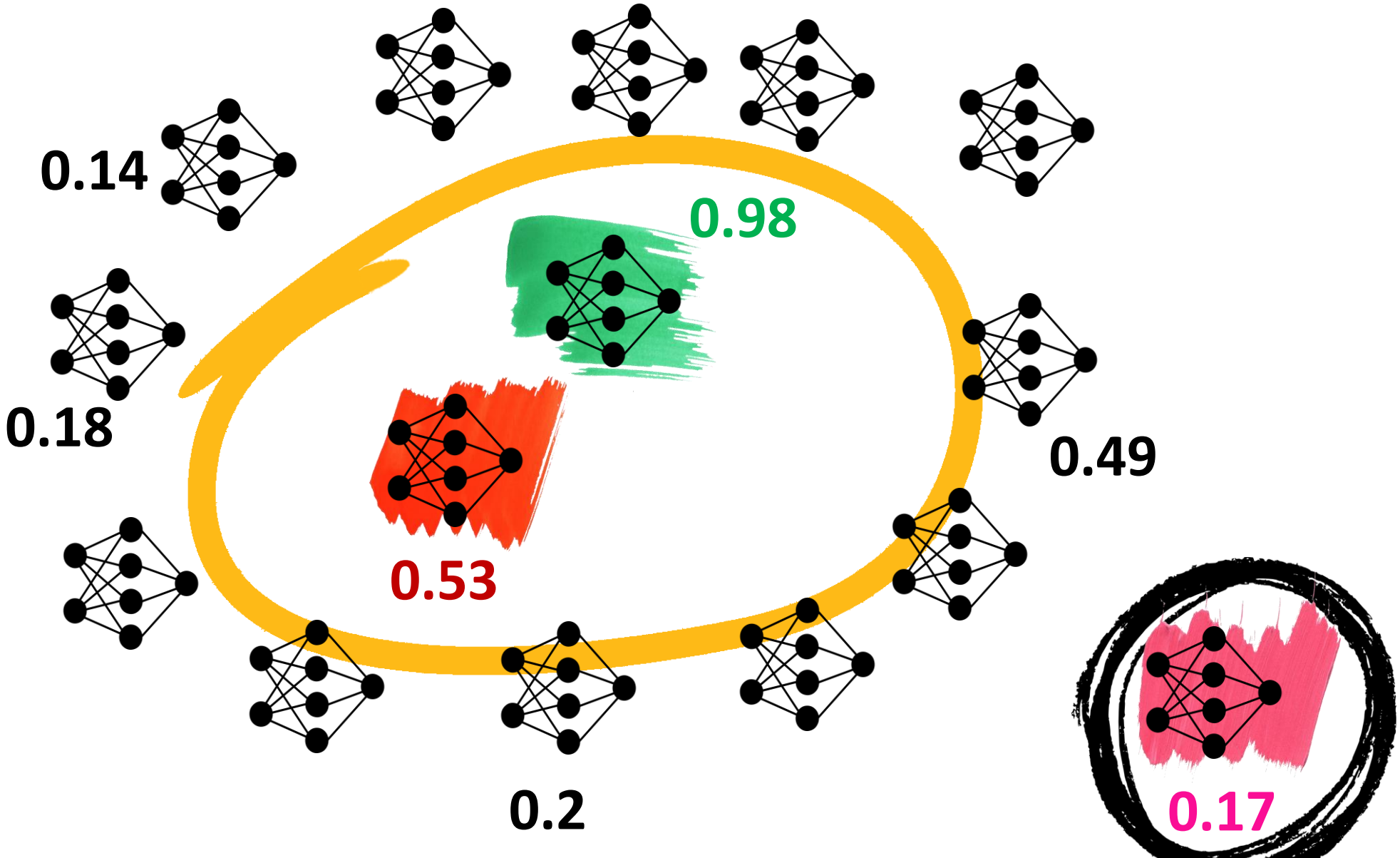
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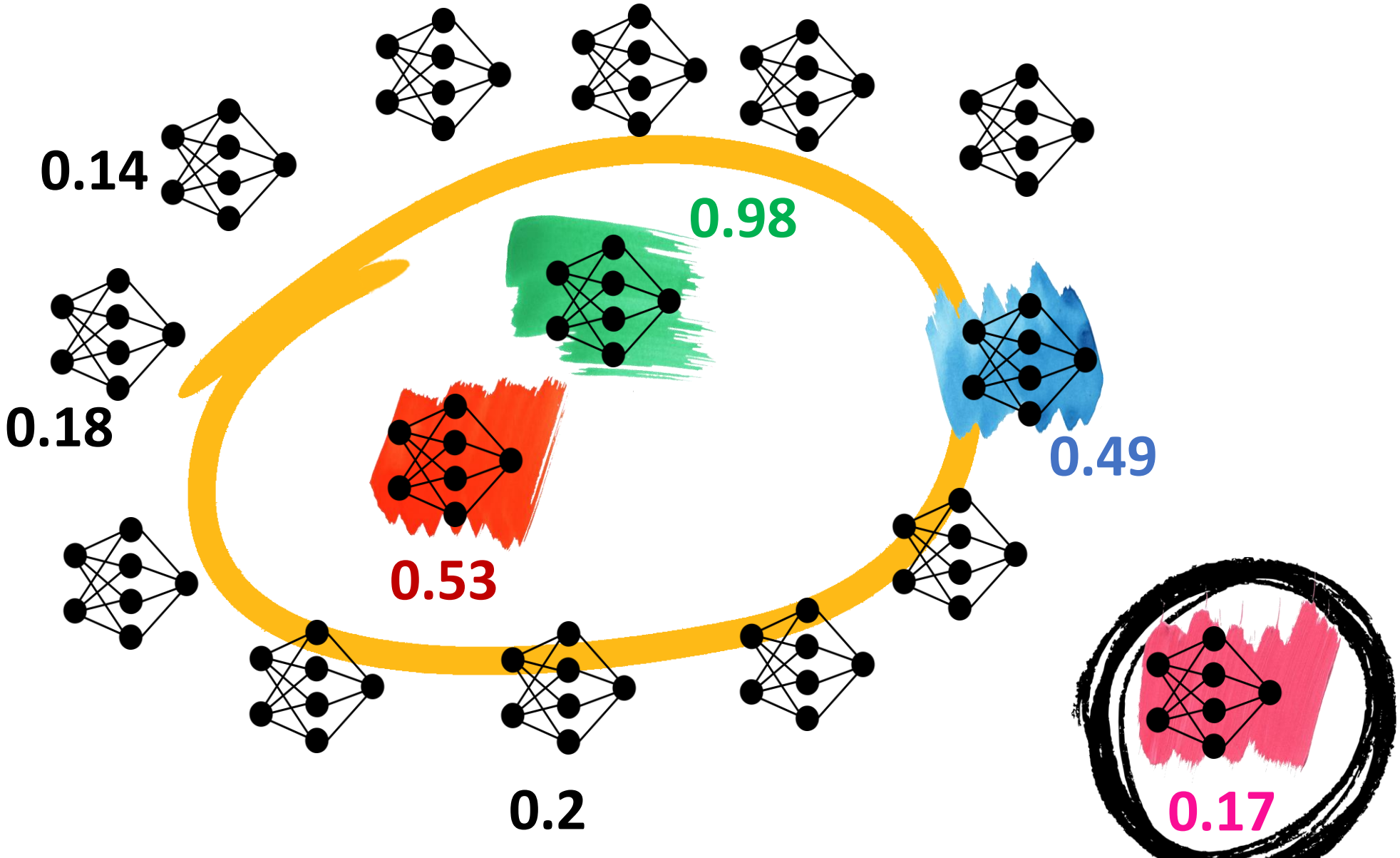
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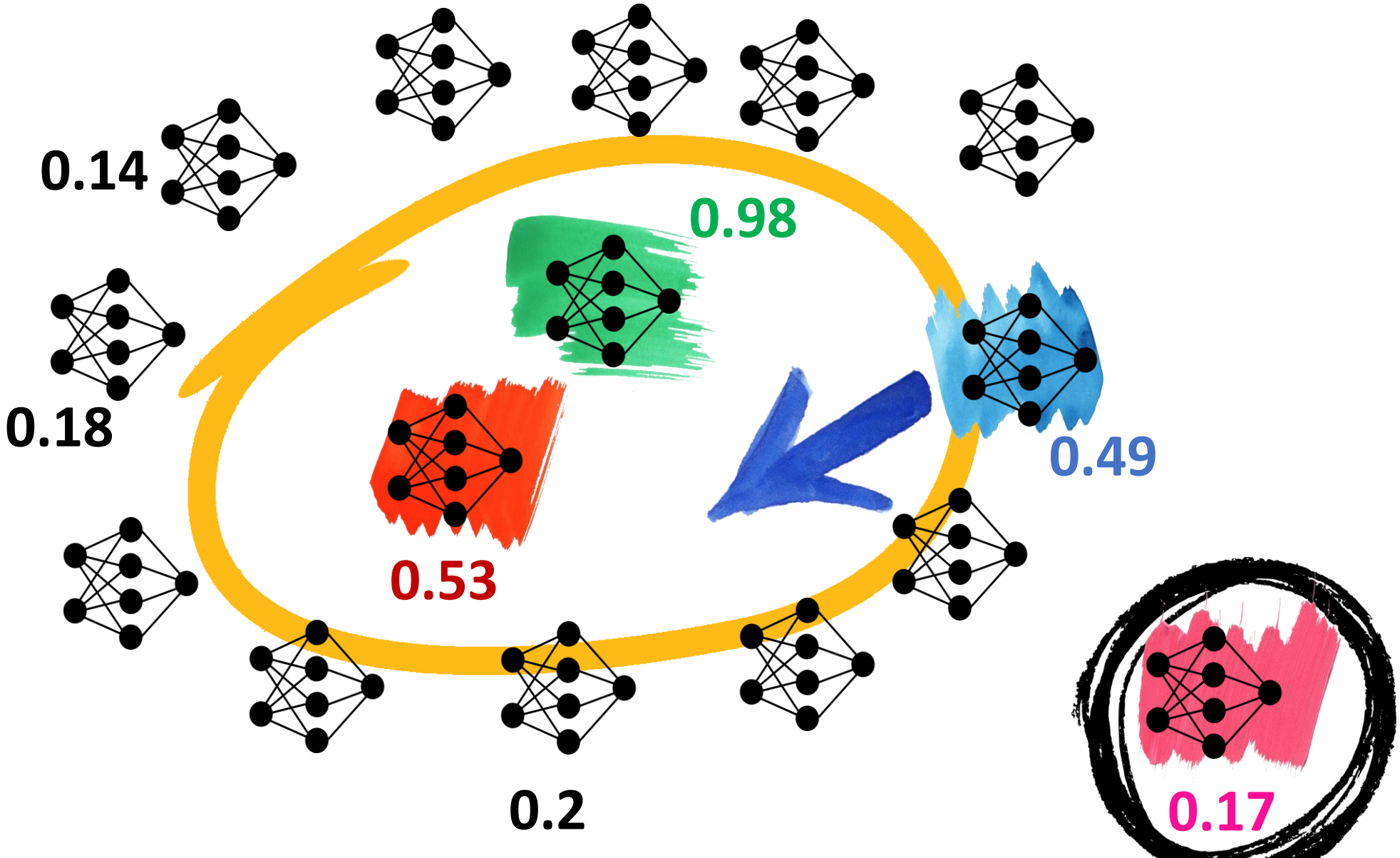
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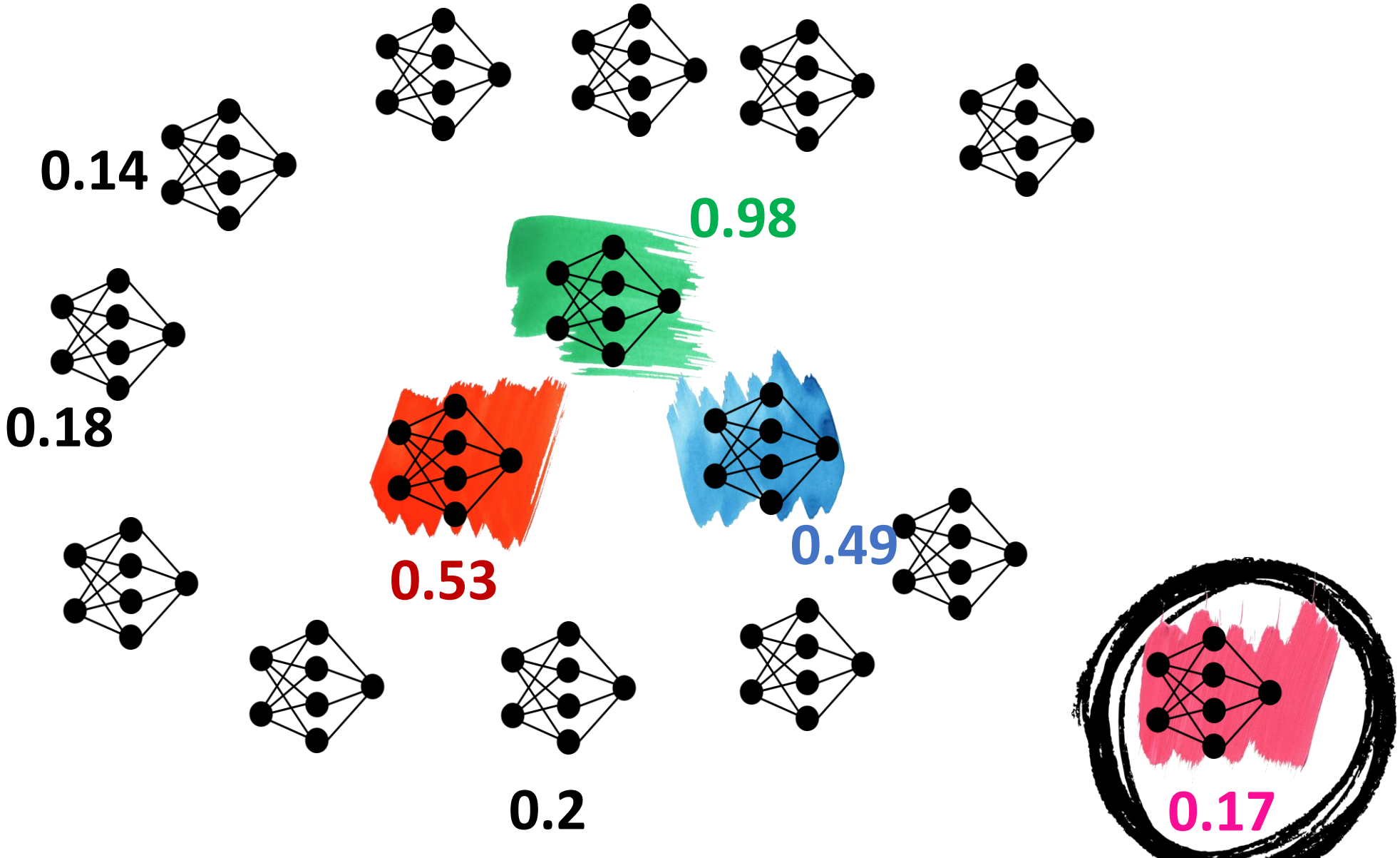
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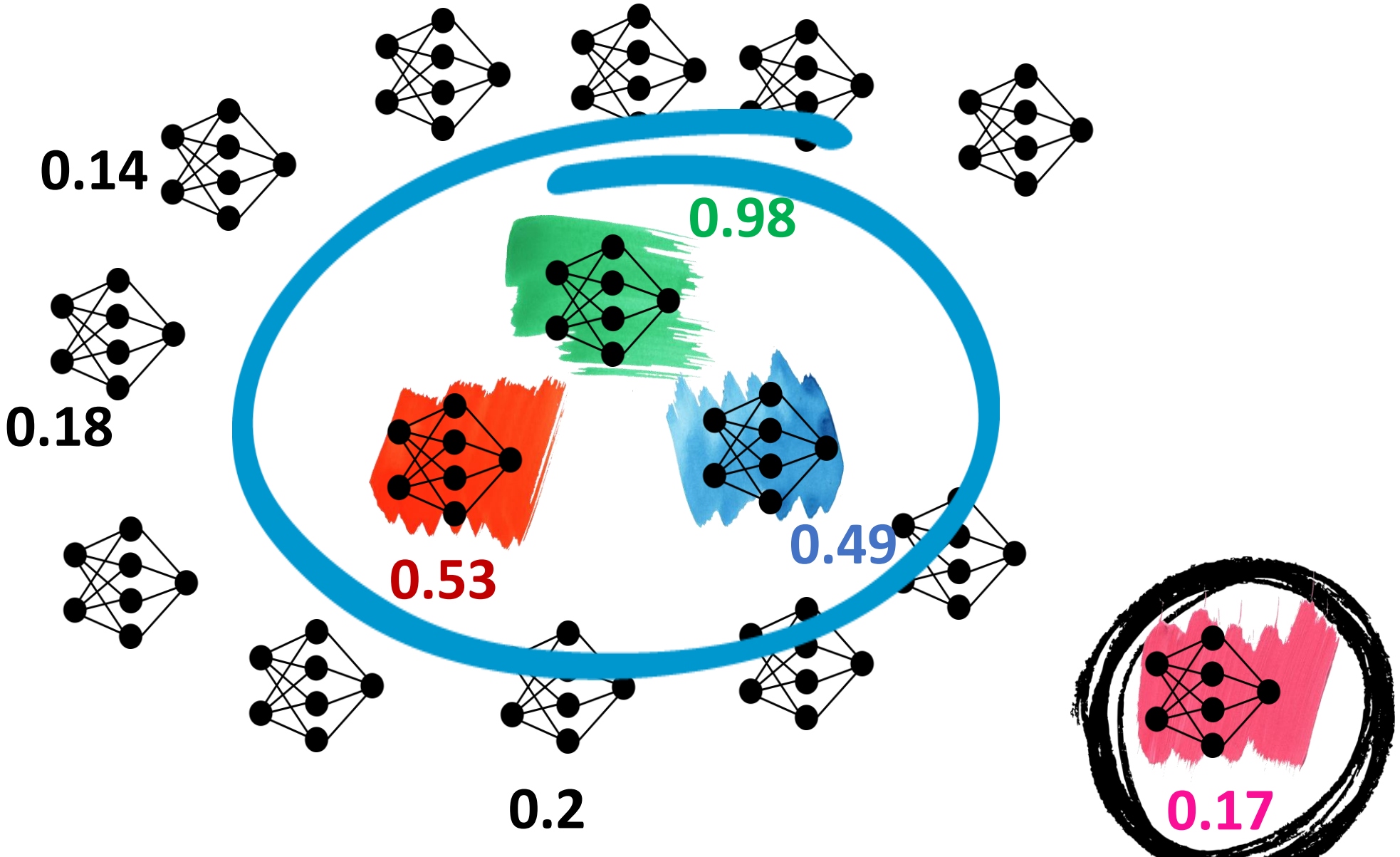
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But..

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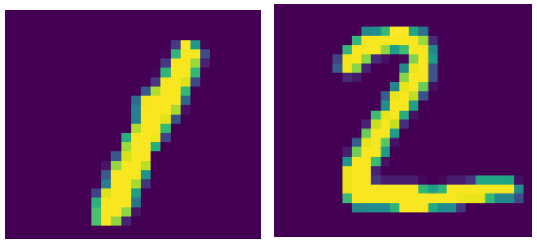
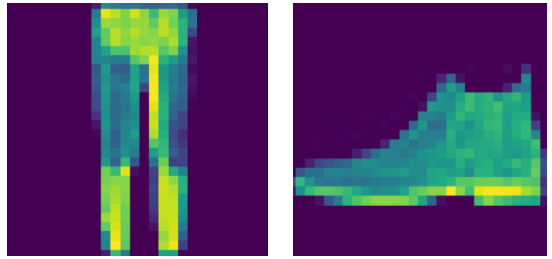
Does it improve *robustness* on non-agreement points?

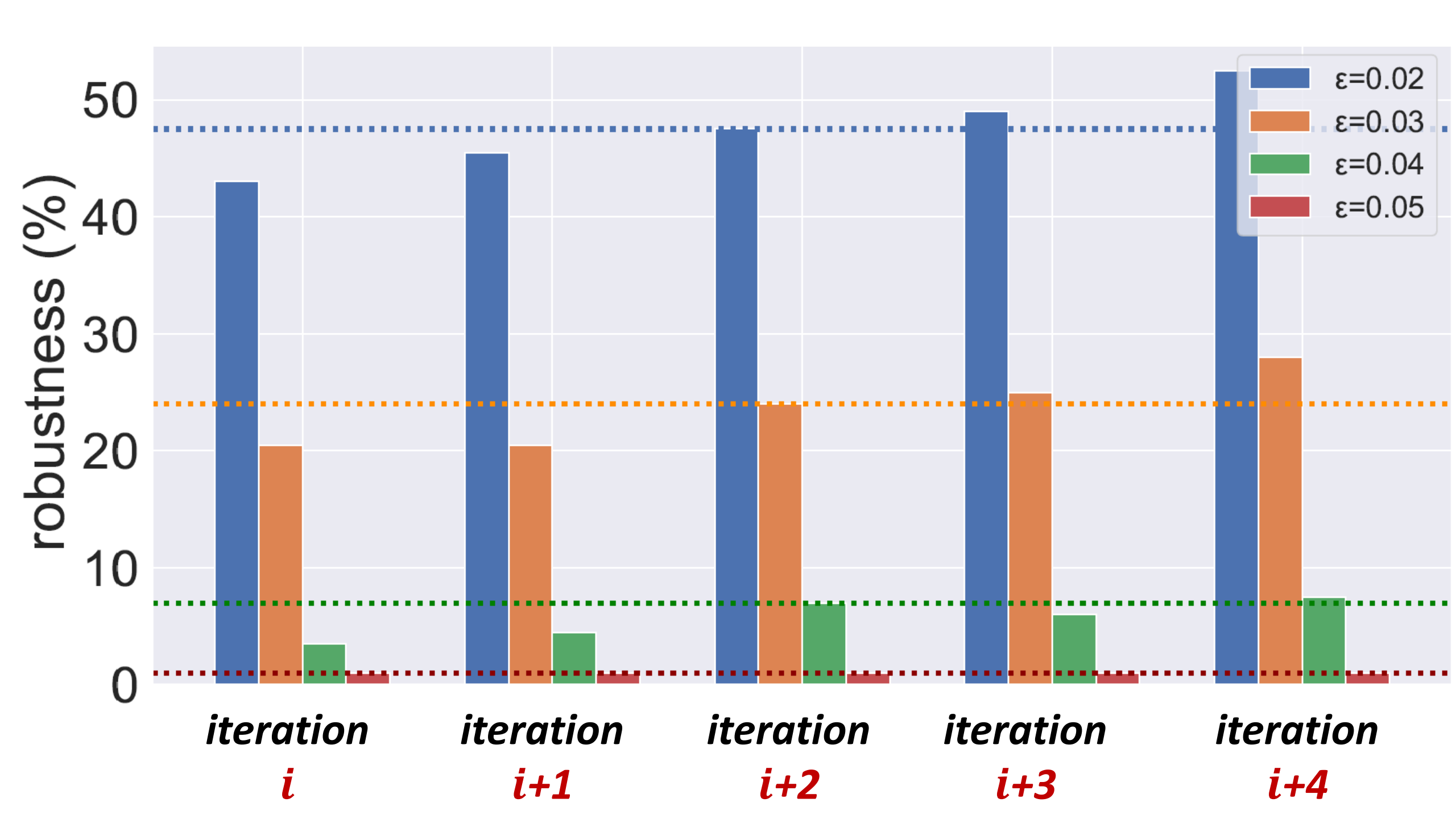
Case Studies

- [x] Train $n = 10$ DNNs $(N_1, \ldots, N_{10})$ on the **MNIST** dataset



- [x] Train $n = 10$ DNNs $(N_{11}, \ldots, N_{20})$ on the **Fashion-MNIST** dataset




robustness (%)
50
40
30
20
10
0
ε=0.02
ε=0.03
ε=0.04
ε=0.05
iteration
i
iteration
i+1
iteration
i+2
iteration
i+3
iteration
i+4

Conclusions

A **greedy-search heuristic** for verification-aided ensemble selection

Using a **polynomial** number of queries

Focusing on **agreement points** (usually) improves robustness on **additional points**

Questions