Synthesizing Locally Symmetric Parameterized Protocols from Temporal Specifications

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FMCAD 2022

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Outline

- **Overview**
- Model of Computation
- Parameterized Synthesis Problem
- Tableau Approach
- Application
- Conclusion and Future Work

Problem

- Synthesize parametric protocols from temporal specifications
  - Locally symmetric protocols composed of many isomorphic copies of a representative process
  - Applications: network communication protocols, distributed algorithms, multi-core hardware models, etc
  - Example: token-passing mutual exclusion
- Undecidable in general [Pnueli & Rosner, 1990]
- State explosion

Our Approach: Global to Local

- Automatically construct a (representative) process P_n
  - P_n is closed under interference by neighboring copies of P_n
  - The closure satisfies a temporal specification φ_n
- Implies that the global structure of any instance $||_i P_i$ satisfies $\bigwedge_i \varphi_i$

Example: Mutual Exclusion

- Goal: prevent simultaneous access to a critical resource
- [Initialization] A single token
- A process uses the token to access the resource and passes the token on completion
- [Interference] Token is passed clockwise between neighboring processes

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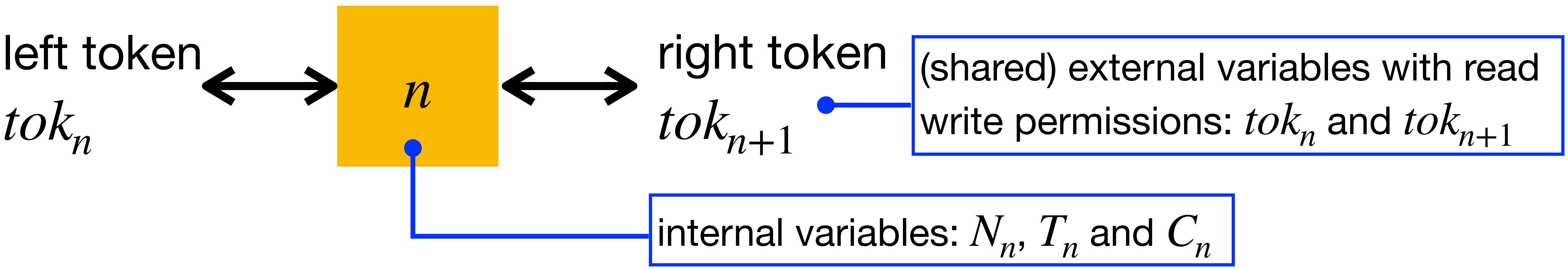
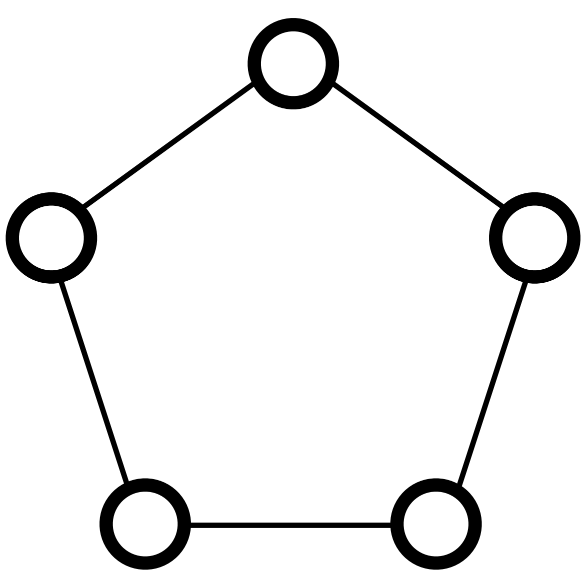
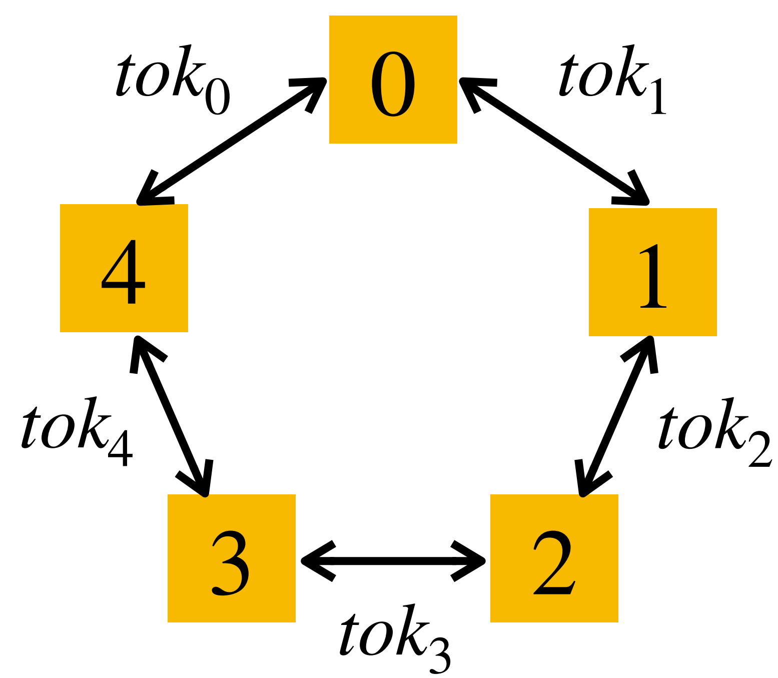
Uniform Ring



The **tile** of a ring-based token-passing mutual exclusion protocol



A ring **architecture** of size 5



An **instance** constructed from the tile

Representative Process

- $P_n = (S_n, S_n^0, T_n, \lambda_n)$
  - A state $s_n \in S_n$ is labeled by internal and external variables
  - For any neighbor m, a joint state is a pair (s_n, t_m)
    - s_n and t_m have the same value for all shared variables
    - e.g., $\left(tok_n, T_n, \neg tok_{n+1} \;,\; \neg tok_{n+1}, N_{n+1}, \neg tok_{n+2} \right)$

Local State Space

- H_n^* is the local state transition system of P_n
- H_n^* has two types of transitions:
  - From s_n to s'_n by n
    - e.g., $tok_n, T_n, \neg tok_{n+1} \xrightarrow{n} tok_n, C_n, \neg tok_{n+1}$
  - [**Interference transitions**] From (s_n, t_m) to (s'_n, t'_m) by neighbors m
    - e.g., $tok_{n-1}, C_{n-1}, \neg tok_n \xrightarrow{n-1} \neg tok_{n-1}, N_{n-1}, tok_n$

Global State Space

- $G = (S, S^0, T, \lambda)$

- Concurrency: nondeterministic interleaving of transitions of processes

- G_n: project out the labels of transitions other than those of n.

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Local Property

- The local property φ_n describes the behavior of P_n in its neighborhood

- Use fair computation tree logic (Fair CTL) to represent φ_n

- Assumption: unconditionally fair scheduling
$\Phi = F^\infty ex_n \wedge \bigwedge_m F^\infty ex_m$

- $\mathsf{A}_\Phi \mathsf{Y}_n$, $\mathsf{E}_\Phi \mathsf{X}_n$, $\mathsf{A}_\Phi \mathsf{G}$, $\mathsf{E}_\Phi \mathsf{G}$, $\mathsf{A}_\Phi \mathsf{F}$, etc

- e.g., a trying process eventually enters into critical
$\mathsf{A}_\Phi \mathsf{G}(T_n \rightarrow \mathsf{A}_\Phi \mathsf{F} C_n)$

Parameterized Synthesis

Theorem II.2. *Let φ_n be a local FairCTL specification. Let P_n be a process such that its derived H_n^* satisfies φ_n. Every instance of the parameterized system constructed from isomorphic copies of P_n satisfies the global property $\bigwedge_i \varphi_i$.*

Procedure:

Write φ_n by hand

↓

Construct H_n^* automatically

↓

Extract P_n from H_n^* automatically

Bisimulation Relation

Theorem II.1. *([NT18]) H_i^* stuttering-simulates G_i for every i. Moreover, if H_i^* satisfies an 'outward-facing' restriction, then H_i^* and G_i are stuttering-bisimular.*

- Assumption: Outward-facing
  - Interference transitions are independent of the internal state of the interfering process

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Tableau Method c.f. [EL86], [EC82]

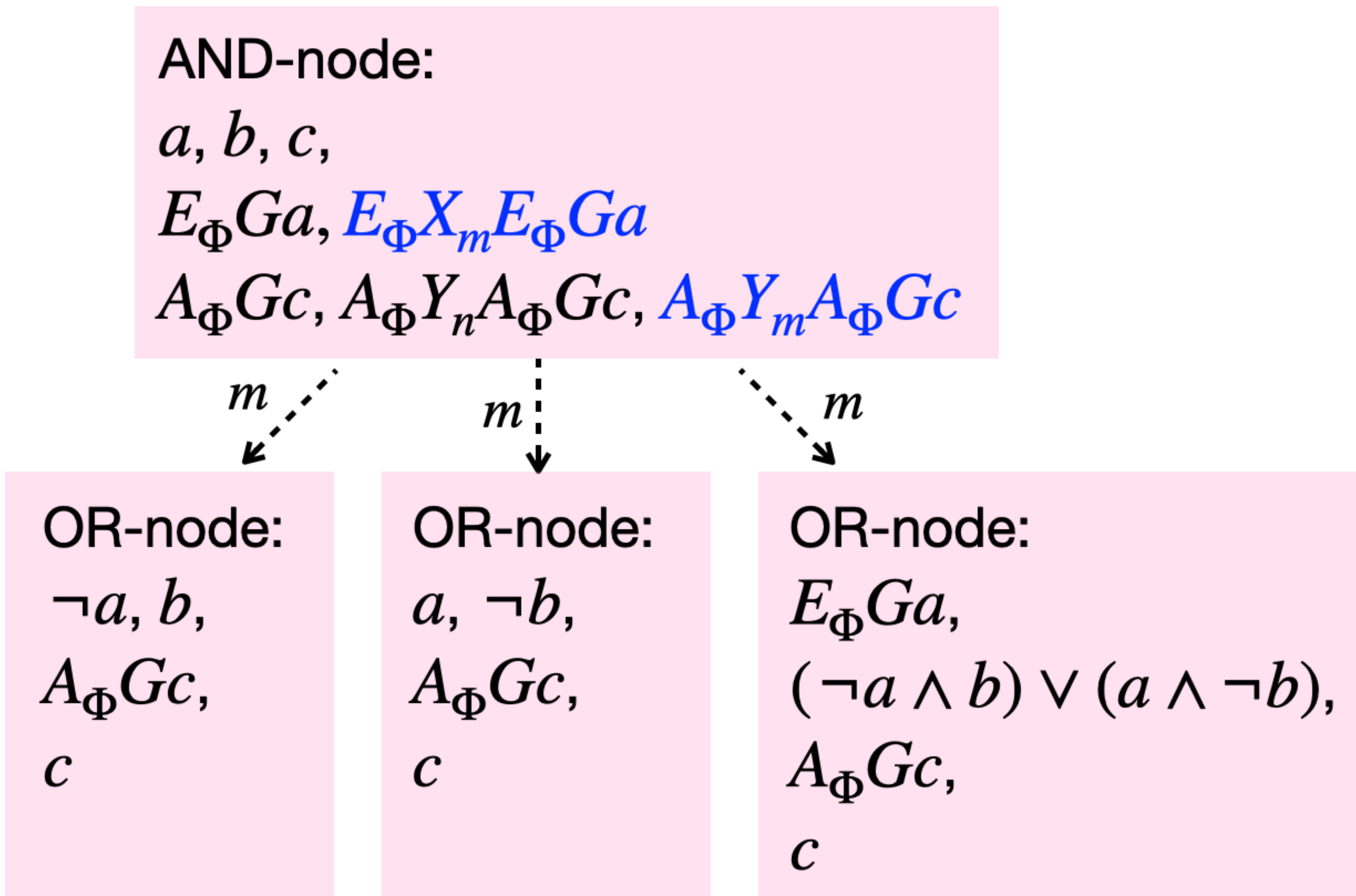
1. Construct the initial tableau $\mathscr{T}_n^0$ from φ_n
2. Construct $\mathscr{T}_n^{i+1}$ from $\mathscr{T}_n^i$
   2.1. Summarize changes by neighbors in $\mathscr{T}_n^i$
   2.2. Add interference transitions
   2.3. Add successors to leaf nodes

repeat, until the tableau reaches its fixpoint

3. Apply deletion rules
4. Extract a model

Interference Transition

Suppose m changes a, b to $\neg a, b$ and $a, \neg b$:



An example of adding interference transitions to an applicable node

Complexity

- The size of the tableau is bounded by $exp(|\varphi_n|)$
  - Local specification with respect to n
- Complexity of synthesizing P_n: polynomial in the size of the tableau c.f. [AAE04]
- Deployment cost: linear in the number of nodes
  - Avoid exponential growth in instance size

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Application

- Mutual exclusion
- Leader election
- Dining philosophers

Mutual Exclusion

Specification:

1. [Initial condition] Exactly one process has the token
2. [Exclusion] No process has multiple tokens

3. [Non-critical] A process moves from non-critical to trying (while keeping the token) or remains in non-critical (while passing the token)
4. [Trying] A process moves from trying to critical with the token
5. [Critical] A process moves from critical to non-critical while passing the token

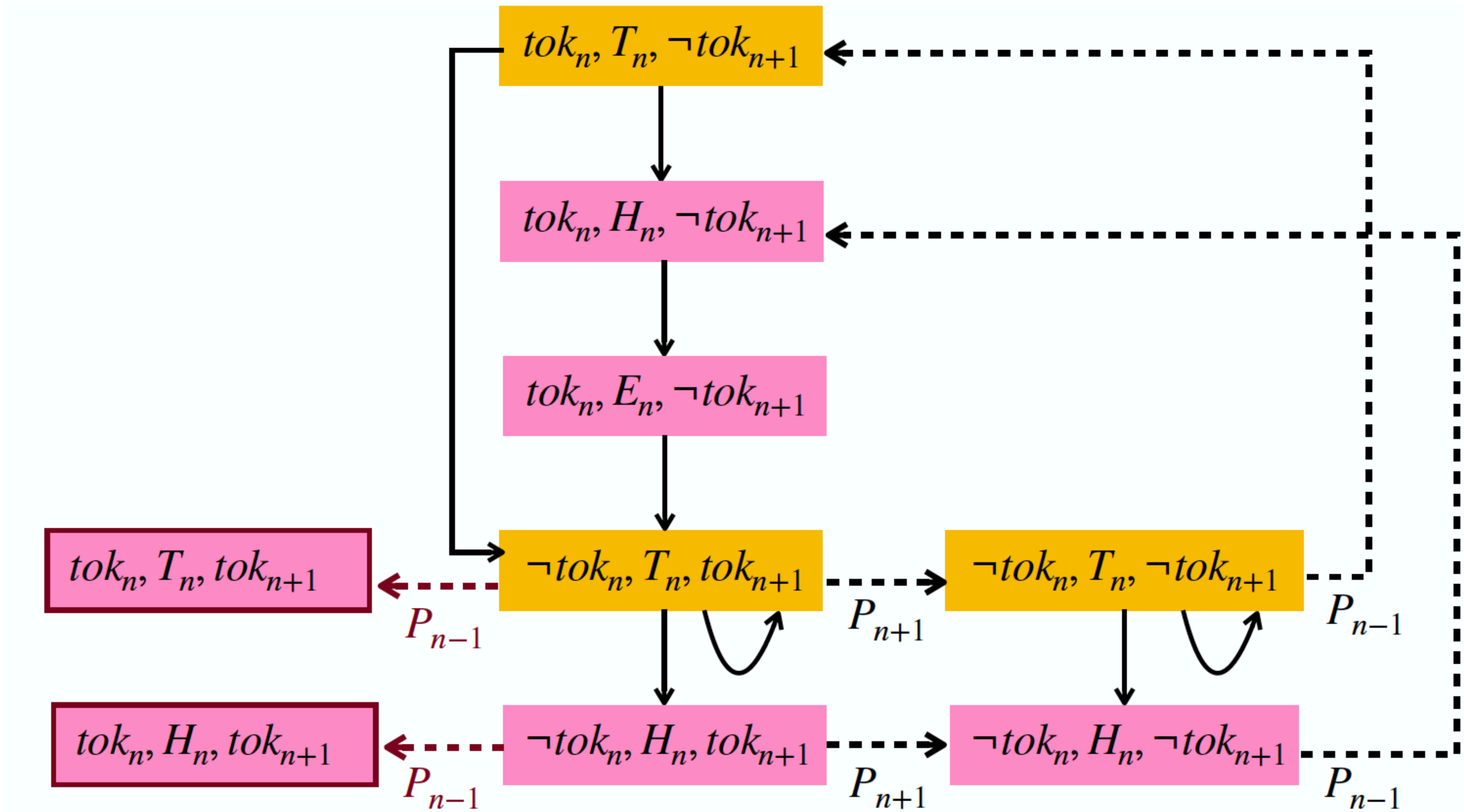
6. [Liveness] A trying process eventually enters into critical
7. [One at a time] A process is in exactly one of the three internal states

Mutual Exclusion

Specification:

1. [Initial condition] $N_n \wedge tok_n \wedge \neg tok_{n+1}$, $N_n \wedge \neg tok_n \wedge tok_{n+1}$, and $N_n \wedge \neg tok_n \wedge \neg tok_{n+1}$
2. [Exclusion] $\mathsf{A}_\Phi \mathsf{G}(\neg tok_n \vee \neg tok_{n+1})$
3. [Non-critical]
   $\mathsf{A}_\Phi \mathsf{G}((N_n \wedge \neg tok_n) \rightarrow (\mathsf{E}_\Phi \mathsf{X}_n(N_n \wedge \neg tok_n) \wedge \mathsf{E}_\Phi \mathsf{X}_n(T_n \wedge \neg tok_n)))$,
   $\mathsf{A}_\Phi \mathsf{G}((N_n \wedge tok_n) \rightarrow (\mathsf{E}_\Phi \mathsf{X}_n(N_n \wedge \neg tok_n \wedge tok_{n+1}) \wedge \mathsf{E}_\Phi \mathsf{X}_n(T_n \wedge tok_n)))$
4. [Trying] $\mathsf{A}_\Phi \mathsf{G}((T_n \wedge tok_n) \rightarrow \mathsf{A}_\Phi \mathsf{Y}_n(C_n \wedge tok_n))$
5. [Critical] $\mathsf{A}_\Phi \mathsf{G}(C_n \rightarrow \mathsf{A}_\Phi \mathsf{Y}_n(N_n \wedge \neg tok_n \wedge tok_{n+1}))$
6. [Liveness] $\mathsf{A}_\Phi \mathsf{G}(T_n \rightarrow \mathsf{A}_\Phi \mathsf{F} C_n)$
7. [One at a time] $\mathsf{A}_\Phi \mathsf{G}(N_n \vee T_n \vee C_n)$, $\mathsf{A}_\Phi \mathsf{G}(N_n \rightarrow (\neg T_n \wedge \neg C_n))$, $\mathsf{A}_\Phi \mathsf{G}(T_n \rightarrow (\neg N_n \wedge \neg C_n))$, and $\mathsf{A}_\Phi \mathsf{G}(C_n \rightarrow (\neg N_n \wedge \neg T_n))$

Mutual Exclusion



The model of φ_n for the mutual exclusion protocol

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Conclusion

- Reduce the global synthesis problem to local
    - Write φ_n by hand
    - From φ_n a representative process P_n is synthesized
    - Representative process can be deployed in instances of arbitrary size

Future Work

- Explore the reduction from global specifications to local
- Apply our approach to other architectures and applications
  - e.g., tori, wrap-around mesh, and other network patterns constructed from tiles
  - e.g., red-black rings, committee coordination, and termination detection

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