Tackling Scalability Issues in Bit-Vector Reasoning

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Satisfiability Modulo Theories (SMT) for Theory of Fixed-Size Bit-Vectors

» **Satisfiability of FOL formula with respect to theories**

- **Focus:** Theory of fixed-size bit-vectors
- Fixed interpretation of theory symbols

Example.

$$(x \ll 001) \geq_s 000 \;\wedge\; x <_u 100 \;\wedge\; (x \cdot 010) \bmod 011 \;=\; x + 001$$

satisfiable with $x := 001$

Theory of Fixed-Size Bit-Vectors

- **bit-vector** as a sequence of bits of a fixed length
  - **constants, variables:** 010, $2_{[3]}$, $x_{[3]}$
  - **bit-vector** operators:
    - **predicates:** $<_u$, $>_s$, ...
    - **bit-wise:** $\sim$, &, |, $\oplus$, ...
    - **shift:** $\ll$, $\gg$, $\gg_a$, rotate
    - **word:** $\circ$, [:], $\langle\rangle_u$, $\langle\rangle_s$, repeat
    - **arithmetic:** $-$, $+$, $\cdot$, $\div$, mod, ...
    - **overflow predicates** for arithmetic operators
- **arithmetic** operators modulo 2^n (**overflow semantics!**)
- **natural representation** for machine integers, hardware registers
- widely used in **hardware and software verification**

Bit-Blasting

- **The State of The Art** for **quantifier-free** BV formulas

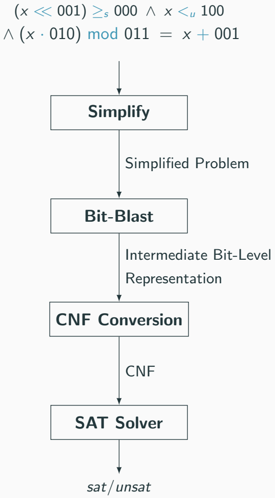
» rewriting + simplifications + **reduction** to SAT

» BV terms » AIG circuit » CNF

- **Main** technique in **state-of-the-art SMT solvers**
  - *Bitwuzla*, *Boolector*, *cvc5*, *MathSAT5*, *STP*, *Yices2*, *Z3*, ...

- **efficient** in practice (**state-of-the-art SAT solvers**)

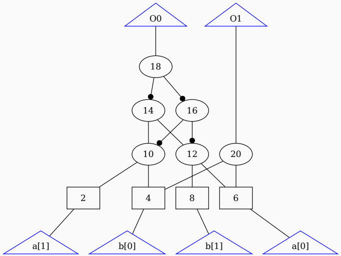
- significant **increase** in formula size



Scalability of Bit-Blasting

- **does not generally scale well** with **increasing** bit-width
- especially with **arithmetic** operators
  - large and complex Boolean circuits
- potential **bottleneck** for SAT solver
  - in practice already as low as 32 bits
- especially severe for applications with **large bit-vectors**
  - **smart contract verification:** 256 bits, heavy use of $\{\cdot, \div, \text{mod}\}$
  - **floating-point arithmetic** when word-blasting
  - **cryptography** (finite fields)

2-bit Multiplier (AIG circuit)



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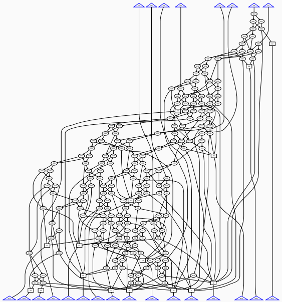
8-bit Multiplier (AIG circuit)



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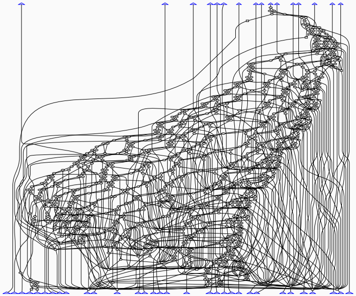
16-bit Multiplier (AIG circuit)



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  - **smart contract verification:** 256 bits, heavy use of $\{\cdot, \div, \text{mod}\}$
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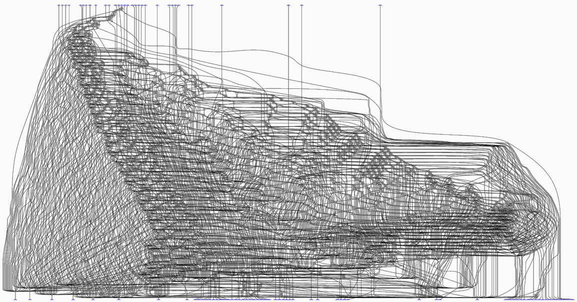
32-bit Multiplier (AIG circuit)



How to tackle these scalability issues?

- **alternative approaches** that do not (mainly) rely on bit-blasting
- **improve scalability** of bit-blasting itself

Alternative Approaches

- **Int-Blasting**: translation to (non-)linear integers
  - Partial int-blasting to linear arithmetic [Bozzano et al. 2006, Rümmer 2008]
  - Full int-blasting to non-linear arithmetic [Zohar et al. 2022]
- **Layered** approach [Bruttomesso et al. 2007, Hadarean et al. 2014]
  - Layer of cheap (but incomplete) procedures, lazy bit-blasting as fallback
- **MCSAT** for bit-vectors [Zeljic et al. 2016, Graham-Lengrand et al. 2020]
  - Word-level explanations for supported fragments, bit-level as fallback
- **Local Search** [Fröhlich et al. 2015, Niemetz et al. 2016, Niemetz et al. 2020]
  - Fast procedures for satisfiable instances
- **PolySAT** [Rath et al. 2024]
  - CDCL(T) solver in Z3 for non-linear bit-vector polynomials

Improving Scalability of Bit-Blasting

- combination of **under- and over-approximation** [Bryant et al. 2007]
  - under-approximation via restricting value ranges of inputs
  - over-approximation via ITE elimination, assertion abstraction and abstracting $x \cdot y$ with partially interpreted function $\lambda x.\lambda y.ite(x \approx 0 \lor y \approx 0, 0, ite(x = 1, y, ite(y \approx 1, x, f(x, y)))$
- similar **under-approximation** techniques in [Brummayer 2009, Jonás et al. 2020]
- **CEGAR-style framework** abstracting $\{\cdot, \div, \text{mod}\}$ [Niemetz et al. 2024]

Every Technique Has Scalability Issues

1,500 benchmarks* instantiated with bit-widths 16,...,8192, ∼ 85 sat, ∼ 1415 unsat

Solved Benchmarks

bw	Bit-Blast Bitwuzla	Lazy+Layered CVC4	MCSAT Yices2	Int-Blast cvc5	PolySAT Z3	VBS
16	1,495					
32	1,459					
64	1,440					
128	1,433					
256	1,388					
512	1,277					
1,024	1,065					
2,048	844					
4,096	816					
8,192	744					
	99% → 49%					

Limits: 1,200 seconds, 8GB memory

* *syrew* benchmarks from [Niemetz et al. 2024]. 500 term and formula equivalence checks enumerated with cvc5's SyGuS solver using SyGuS grammar $\{0, 1, x, s, t, \approx, \not\approx, <_u, \leq_u, \sim, \&, \ll, \gg, \diamond\}$ for $\diamond \in \{\cdot, \div, \text{mod}\}$.

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64	1,440	1,368				
128	1,433	1,308				
256	1,388	1,232				
512	1,277	1,162				
1,024	1,065	774				
2,048	844	401				
4,096	816	300				
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	99% $\rightarrow$ 49%	97% $\rightarrow$ 13%				

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32	1,459	1,390	1,194			
64	1,440	1,368	1,112			
128	1,433	1,308	1,076			
256	1,388	1,232	987			
512	1,277	1,162	916			
1,024	1,065	774	794			
2,048	844	401	668			
4,096	816	300	572			
8,192	744	202	492			
	99% → 49%	97% → 13%	93% → 33%			

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32	1,459	1,390	1,194	1,102		
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128	1,433	1,308	1,076	1,017		
256	1,388	1,232	987	916		
512	1,277	1,162	916	788		
1,024	1,065	774	794	613		
2,048	844	401	668	528		
4,096	816	300	572	428		
8,192	744	202	492	389		
	99% → 49%	97% → 13%	93% → 33%	74% → 26%		

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32	1,459	1,390	1,194	1,102	672	
64	1,440	1,368	1,112	1,077	668	
128	1,433	1,308	1,076	1,017	648	
256	1,388	1,232	987	916	637	
512	1,277	1,162	916	788	620	
1,024	1,065	774	794	613	608	
2,048	844	401	668	528	576	
4,096	816	300	572	428	562	
8,192	744	202	492	389	552	
	99% → 49%	97% → 13%	93% → 33%	74% → 26%	46% → 37%	

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2,048	844	401	668	528	576	**1,133**
4,096	816	300	572	428	562	**1,074**
8,192	744	202	492	389	552	**993**
	99% → 49%	97% → 13%	93% → 33%	74% → 26%	46% → 37%	**100% → 66%**

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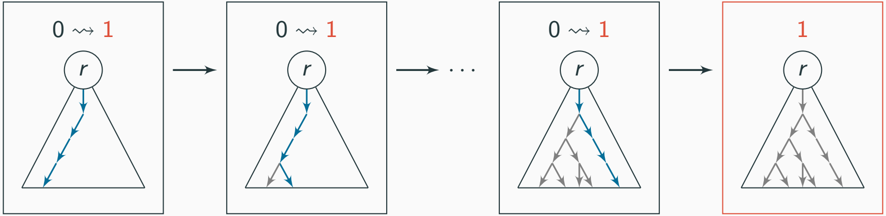
This Talk

- **alternative approaches** that do not (mainly) rely on bit-blasting
  - **Ternary Propagation-Based Local Search** [Niemetz et al. 2020]
    - fast procedures for satisfiable instances
    - without bit-blasting
    - propagation via invertibility conditions
- **improve scalability** of bit-blasting itself
  - **CEGAR-style framework** abstracting $\{\cdot, \div, \text{mod}\}$ [Niemetz et al. 2024]
    - avoid bit-blasting of expensive operators
    - bit-blasting of abstracted terms only as last resort
    - refinement schemes utilize invertibility conditions

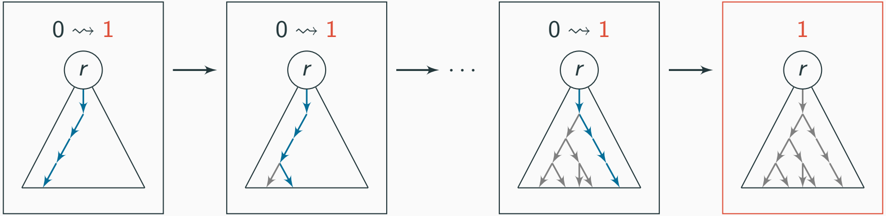
Ternary Propagation-Based Local Search*



- **alternative** to bit-blasting
- lifts concept of backtracing from ATPG to the **word-level**
- cannot determine **unsatisfiability**
- non-deterministic but **probabilistically approximately complete (PAC)** [Hoos 1999]
  - » guaranteed to find a solution if there is one

* [Niemetz et al. 2016]◇⋆ [Niemetz et al. 2020]⋆ joint work with A. Biere◇, M. Preiner⋆

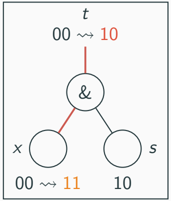
Ternary Propagation-Based Local Search



1. assume satisfiability, start with **initial assignment**
2. **propagate** target values along a single path towards inputs
   - invertibility (conditions)
3. iteratively improve current state until **solution** is found

- **main** sources of **non-determinism**:
  - propagation path selection
  - propagation value selection

Value Propagation via Invertibility Conditions



inverse value

yields target value immediately

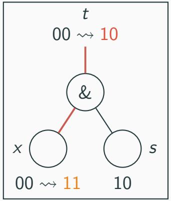
Invertibility Condition (IC)

Given $x \diamond s \approx t$, is there a value for x given s and t?

» $\forall x, s, t.\ IC(x, x \diamond s \approx t) \Leftrightarrow \exists y.\ y \diamond s \approx t$

Example. $IC(x, x \,\&\, s \approx t) = t \,\&\, s \approx t$

Value Propagation via Invertibility Conditions



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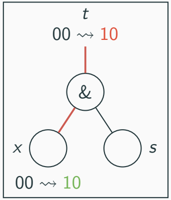

consistent value

yields target value
after changing other inputs

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Given $x \diamond s \approx t$, is there a value for x given s and t?

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Example. $IC(x, x \,\&\, s \approx t) = t \,\&\, s \approx t$

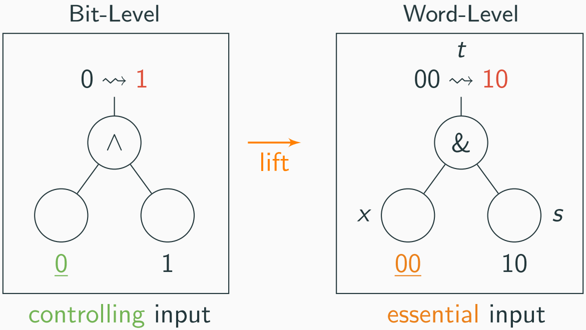
Consistency Condition (CC)

Given $x \diamond s \approx t$, is there a value for x given t?

» $\forall x, t.\ CC(x, x \diamond x \approx)t \Leftrightarrow \exists y, s.\ y \diamond s \approx t$

Example. $IC(x, x \,\&\, s \approx t) = \top$

Propagation Path Selection



- x is **essential** if $IC(s, x \diamond s \approx t) = \bot$
- s is **essential** if $IC(x, x \diamond s \approx t) = \bot$

▶ prefer **essential** input (if any)

Tackling Scalability Issues in Ternary Propagation-Based Local Search

- **complementary** to bit-blasting
- **scalability** issues with **increasing bit-width**
  - **too many** possible candidates for value selection
  - especially for disequality, inequalities, bit-wise operators
- utilize **constant bit** information
- respect **bounds** implied from top-level constraints

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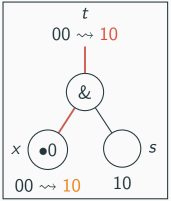

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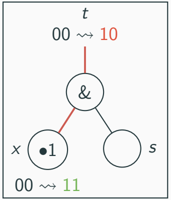
Example. $IC(x, x \,\&\, s \approx t) = t \,\&\, s \approx t$
$\wedge ((s \,\&\, x^{hi}) \,\&\, \sim(x^{lo} \oplus x^{hi})) \approx (t \,\&\, \sim(x^{lo} \oplus x^{hi}))$



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- **const bits** represented as **ternary bit-vector** $x = \langle x^{lo}, x^{hi} \rangle$

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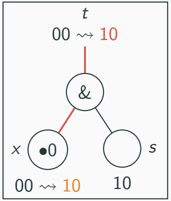

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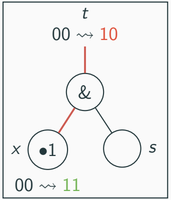

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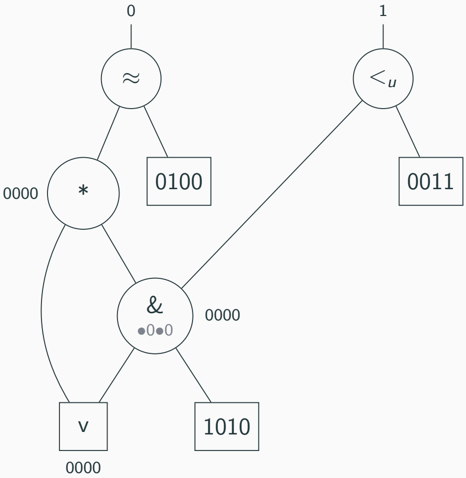
Example. $v_{[4]} \cdot (v_{[4]} \mathbin{\&} 1010) \approx 0100 \wedge (v_{[4]} \mathbin{\&} 1010) <_u 0011$



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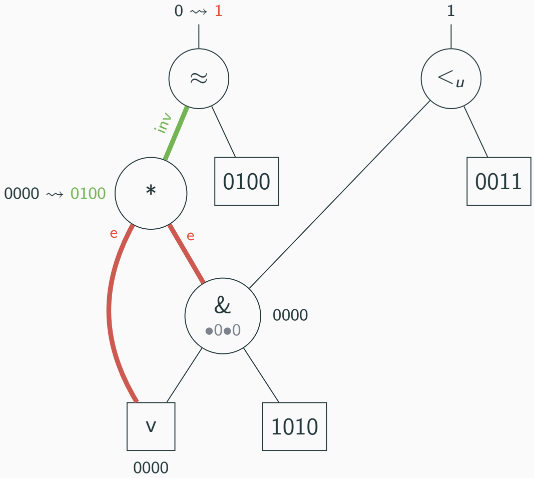

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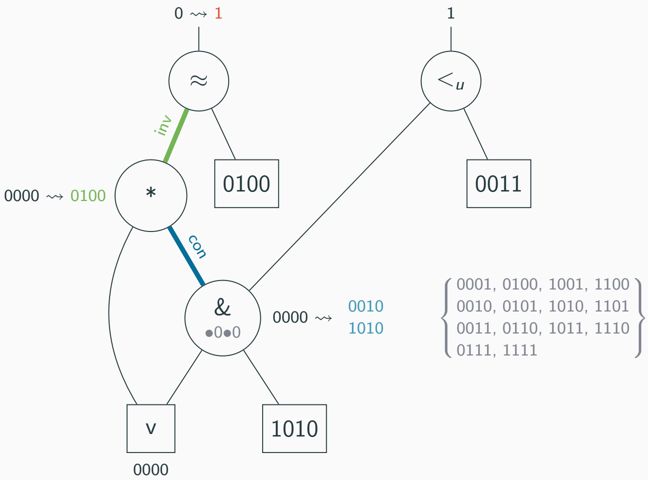

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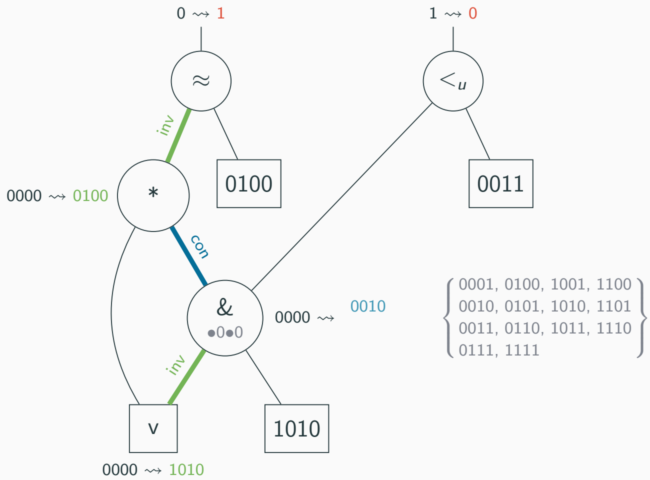

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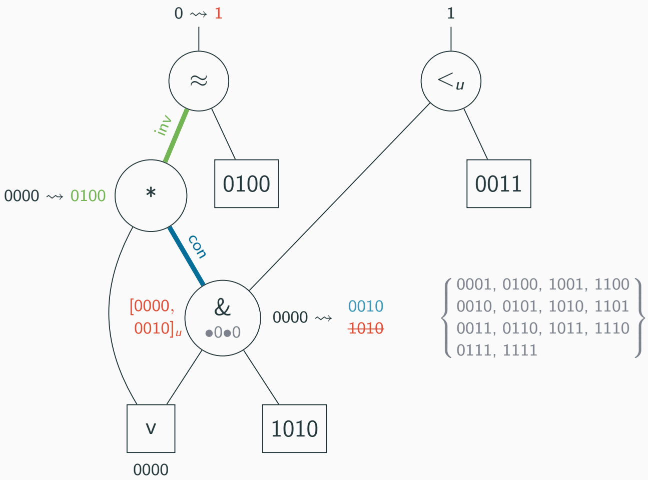

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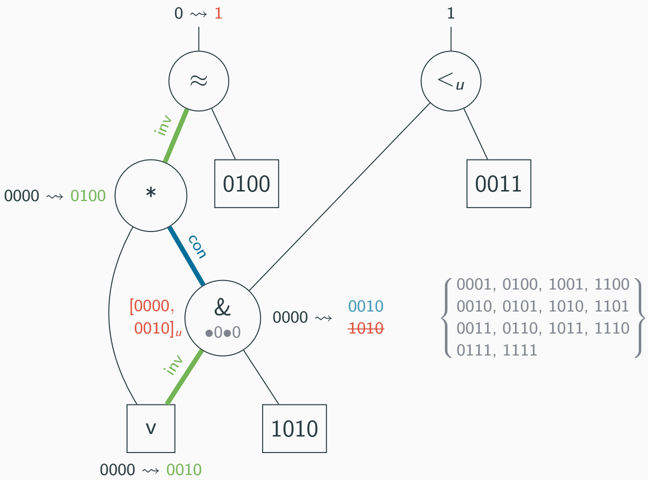

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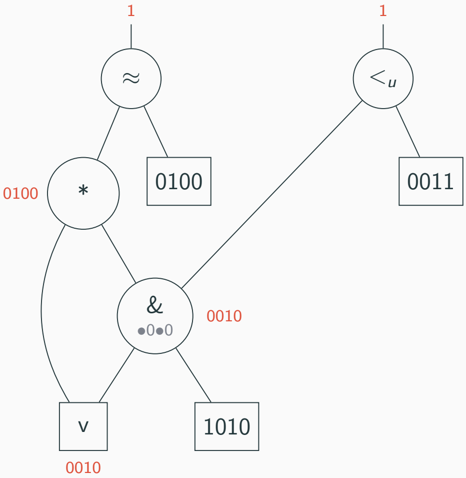


Invertibility Conditions

- **symbolic invertibility and consistency conditions**
  - synthesized manually and with Syntax-Guided Synthesis (SyGuS)
  - **w/o const bits:** 50% with SyGuS (only IC)
    - » formally proved correct [Niemetz et al. 2021, Ekici et al. 2023]
  - **with const bits:** 5% with SyGuS (IC + CC)
    - » verified up to size 65, formal correctness proof future work

Beyond Local Search

- Quantifier instantiation [Niemetz et al. 2018]
  - » default approach for BV in cvc5 (SMT-COMP winner in BV-SQ 2018, 2020–2024)
- Floating-Point Arithmetic [Brain et al. 2019a]
  - » invertibility conditions for majority of FP operators
- **Bit-Vector Abstraction** [Niemetz et al. 2024]
  - » **CEGAR-style framework** abstracting $\{\cdot, \div, \text{mod}\}$

Scalable Bit-Blasting with Abstractions*

- **improve scalability** of bit-blasting itself

- a **CEGAR-style** abstraction-refinement framework for **theory BV**
  - based on **bit-blasting**
  - implemented in our SMT solver **Bitwuzla**
  - significantly **improves** performance on **large** bit-vectors

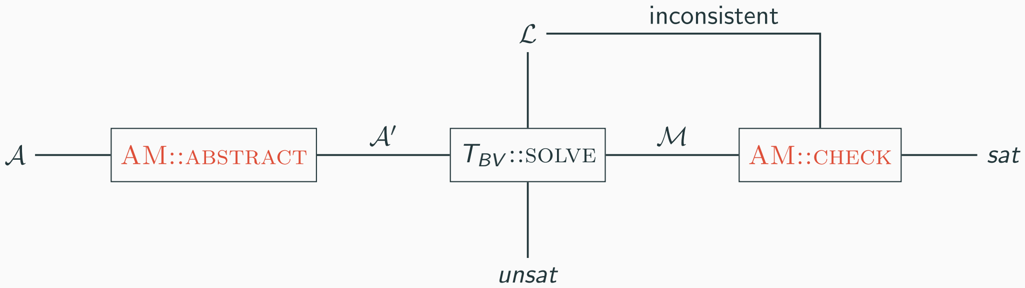
- **abstraction refinement scheme** for bit-vector operators $\{\cdot, \div, \text{mod}\}$
  - **most expensive** operators for bit-blasting
  - 70 lemmas total

- abduction-based framework for **lemma synthesis**

- lemma **scoring** scheme

* [Niemetz et al. 2024] joint work with M. Preiner and Y. Zohar

Abstraction-Refinement Loop

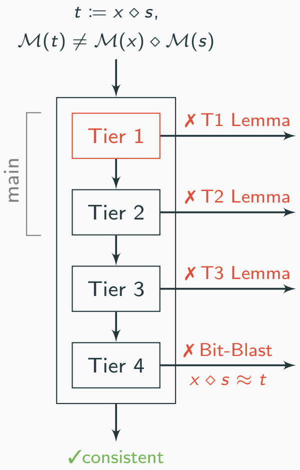


- abstract each relevant $\{\cdot, \div, \text{mod}\}$ term with **fresh constant**
- **over-approximation**

- **check consistency** with true semantics of operators
  - » consistent: ✓
  - » inconsistent: refine abstraction

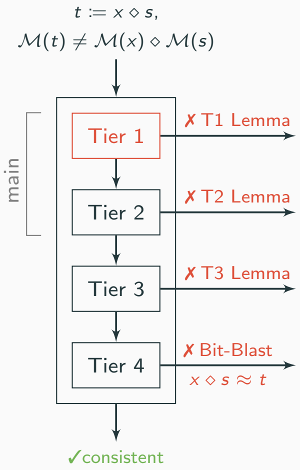
4-Tiered Refinement Scheme

- **CEGAR Loop Refinement Step for Abstracted Term**
  - tiers processed in order 1-4
  - if a lemma is violated, use as refinement



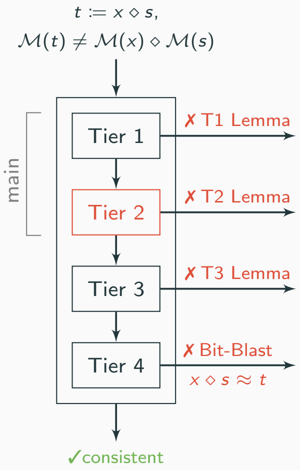
4-Tiered Refinement Scheme

- **Tier 1 Hand-Crafted Lemmas**
  - properties described by **invertibility condition** IC
    - $\exists x.\, x \diamond s \approx t \Leftrightarrow IC[s,t]$
    - e.g., $x \cdot s$ has at least as many trailing zeros as x or s
      - » $((-s \mid s)\ \&\ t) \approx t$
      - » $((-x \mid x)\ \&\ t) \approx t$
  - **basic** properties of the abstracted operator
    - e.g., abstraction t for $x \cdot s$ is odd iff x and s are odd
      - » $t[0] \approx (x[0]\ \&\ s[0])$
  - **verified** up to bit-width 512 (17 lemmas)



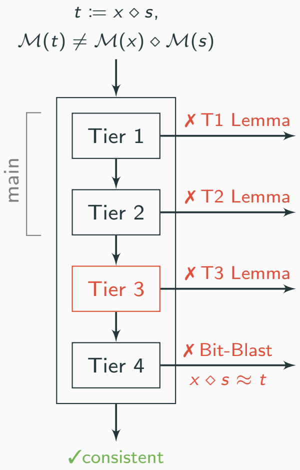
4-Tiered Refinement Scheme

- **Tier 2 Synthesized Lemmas**
  - synthesized via **syntax-restricted abduction** with **cvc5** (offline)
    - **abduction:** A, B, find C s.t. $A \wedge C \Rightarrow B$ valid, $A \wedge C$ sat
    - find **lemma** ℓ such that $\top \wedge \neg\ell \Rightarrow x \diamond s \not\approx t$
  - e.g., for $x \cdot s \approx t$
    - $x \not\approx (1 \oplus (x \ll (s \oplus t)))$
  - lemmas **filtered** based on **lemma score**
  - **with respect to hand-crafted (tier 1) lemmas**
  - **verified** up to bit-width 512 (53 lemmas)



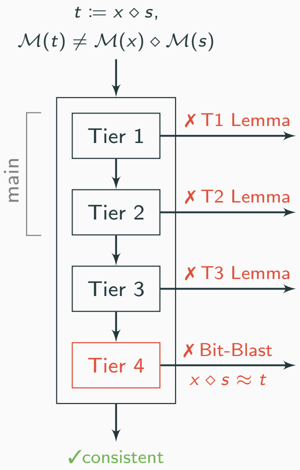
4-Tiered Refinement Scheme

- **Tier 3** **Value Instantiation** Lemmas
  - rule out **current inconsistent model value**
  - only added if none in tiers 1–2 are violated
  - **limited** fallback strategy
    - limited to $w/8$ instantiations per term
    - e.g., 4 instantiations for 32 bit
  - e.g., for $x \cdot s$ and $\mathcal{M} = \{x_{[32]} = 3, s_{[32]} = 6, t_{[32]} = 1\}$,
    we add lemma $(x = 3 \wedge s = 6) \Rightarrow t = 18$



4-Tiered Refinement Scheme

- **Tier 4** **Bit-blasting** lemmas
  - **last resort**
  - add lemma to **enforce bit-blasting** of the abstracted term
  - e.g., lemma $t \approx x \cdot s$ for $x \cdot s$



Tier 1* and 2 Refinement Lemmas ($t := x \diamond s$)

`bvmul`			
1^*	$s \approx 2^i \Rightarrow t \approx x \ll i$	$11_{>1}$	$t \not\approx (1 \mid {\sim}(x \oplus s))$
2^*	$s \approx -2^i \Rightarrow t \approx -x \ll i$	$12_{>1}$	$t \not\approx ({\sim}1 \mid (x \oplus s))$
3^*	$((-s \mid s) \,\&\, t) \approx t$	13	$x \not\approx ((x \ll (s+t)) - 1)$
4^*	$t[0] \approx (x[0] \,\&\, s[0])$	14	$x \not\approx (1 - (x \ll (s-t)))$
$5_{>1}$	$s \not\approx {\sim}(t \mid (1 \,\&\, (x \mid s)))$	15	$s \not\approx (1 + (s \ll (t-x)))$
$6_{>1}$	$(x \,\&\, t) \not\approx (s \mid {\sim}t)$	16	$s \not\approx (1 - (s \ll (t-x)))$
$7_{>1}$	$t \not\approx ((s \mid 1) \ll (t \ll x))$	17	$s \not\approx (1 + (s \ll (x-t)))$
8	$s \approx (s \ll (x \,\&\, (1 \gg t)))$	$18_{>1}$	$t \not\approx (1 \mid (x+s))$
$9_{\neq 2}$	$t \geq_u (1 \,\&\, ((x \,\&\, s) \gg 1))$	19	$x \not\approx {\sim}(x \ll (s+t))$
10	$x \not\approx (1 \oplus (x \ll (s \oplus t)))$		

`bvurem`			
1^*	$s \approx 2^i \Rightarrow t \approx (0_{[\kappa(x)-i]} \circ x[i-1:0])$	9	$x \geq_u (t \mid (x \,\&\, s))$
2^*	$s \not\approx 0 \Rightarrow t \leq_u s$	10	$1 \not\approx (t \,\&\, {\sim}(x \mid s))$
3^*	$x \approx 0 \Rightarrow t \approx 0$	11	$t \not\approx ({\sim}x \mid -s)$
4^*	$s \approx 0 \Rightarrow t \approx x$	12	$(t \,\&\, (x \mid s)) \geq_u (t \,\&\, 1)$
5^*	$s \approx x \Rightarrow t \approx 0$	$13_{>2}$	$x \not\approx (-x \mid -{\sim}t)$
6^*	$x <_u s \Rightarrow t \approx x$	14	$(x + -s) \geq_u t$
7^*	${\sim}-s \geq_u t$	15	$(-s \oplus (x \mid s)) \geq_u t$
8	$x \approx (x \,\&\, (s \mid (t \mid -s)))$		

`bvudiv`			
1^*	$s \approx 2^i \Rightarrow t \approx x \gg i$	19	$(x \gg t) \not\approx (s \mid t)$
2^*	$(s \approx x \wedge s \not\approx 0) \Rightarrow t \approx 1$	20	$s \not\approx {\sim}(s \gg (t \gg 1))$
3^*	$s \approx 0 \Rightarrow t \approx {\sim}0$	$21_{>1}$	$x \not\approx {\sim}(x \,\&\, (t \ll 1))$
4^*	$(x \approx 0 \wedge s \not\approx 0) \Rightarrow t \approx 0$	22	$t \geq_u ((x \ll 1) \gg s)$
5^*	$s \not\approx 0 \Rightarrow t \leq_u x$	23	$x \geq_u (s \ll {\sim}(x \mid t))$
6^*	$(s \approx {\sim}0 \wedge x \not\approx {\sim}0) \Rightarrow t \approx 0$	24	$x \geq_u (t \ll {\sim}(x \mid s))$
7	$x \geq_u -(-s \,\&\, -t)$	25	$x \geq_u (t \oplus (t \gg (s \gg 1)))$
8	$-(s \mid 1) \geq_u t$	26	$x \geq_u (s \oplus (s \gg (t \gg 1)))$
9	$t \not\approx -(s \,\&\, {\sim}x)$	27	$x \geq_u (s \ll {\sim}(x \oplus t))$
10	$(s \mid t) \not\approx (x \,\&\, {\sim}1)$	28	$x \geq_u (t \ll {\sim}(x \oplus s))$
11	$(s \mid 1) \not\approx (x \,\&\, {\sim}t)$	29	$x \not\approx (t + (s \mid (x+s)))$
12	$(x \,\&\, -t) \geq_u (s \,\&\, t)$	$30_{>2}$	$x \not\approx (t + (1 + (1 \ll x)))$
13	$s \geq_u (x \gg t)$	31	$s \geq_u ((x+t) \gg t)$
14	$x \geq_u ((s \gg (s \ll t)) \ll 1)$	$32_{>1}$	$x \not\approx (t + (t + (x \mid s)))$
15	$x \geq_u ((t \ll 1) \gg (t \ll s))$	33	$(s \oplus (x \mid t)) \geq_u (t \oplus 1)$
16	$t \geq_u ((x \gg s) \ll 1)$	34	$t \geq_u (x \gg (s-1))$
17	$x \geq_u ((x \mid t) \,\&\, (s \ll 1))$	35	$(s-1) \geq_u (x \gg t)$
18	$x \geq_u ((x \mid s) \,\&\, (t \ll 1))$	$36_{\neq 2}$	$x \not\approx (1 - (x \ll (x-t)))$

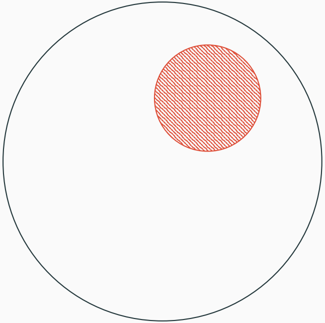
Lemma Score

- **Metric for quality of a lemma**

- Given abstracted term $x \diamond s$ and lemma $\ell[x, s, t]$ s.t. $x \diamond s \approx t \Rightarrow \ell$, then
  $\textsc{Score}(\ell, w) :=$ num. triplets (v^x, v^s, v^t) where $\ell[v^x, v^s, v^t] = \top$.

Example. $x \cdot s$ with $w = 4$

- Worst score: $2^4 \times 2^4 \times 2^4 = 4096$ $(\ell = \top)$
- Best score: $2^4 \times 2^4 = 256$ $(\ell = x \cdot s \approx t)$



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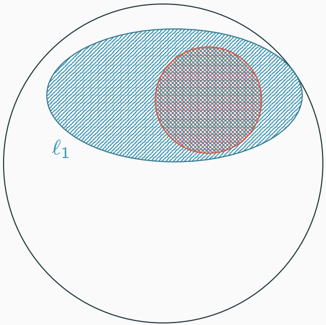

ℓ_1: 2416

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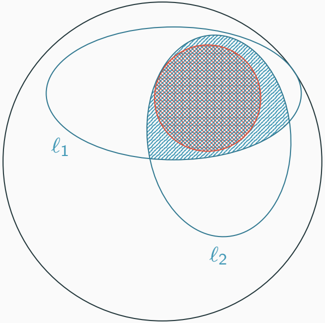

ℓ_1: 2416

ℓ_2: 2791

Lemma Score

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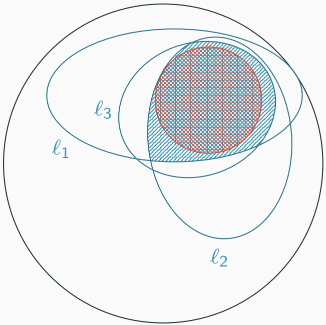

ℓ_1: 2416
ℓ_2: 2791
ℓ_3: 1961

Lemma Score

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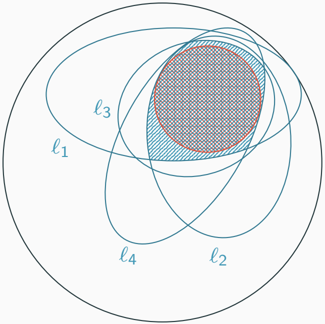
- Best score: $2^4 \times 2^4 = 256$ $(\ell = x \cdot s \approx t)$
- **Score for hand-crafted $\{\ell_1, \ldots, \ell_4\}$: 704**
  - » **rule out 88% of incorrect triplets**



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ℓ_4: 2048

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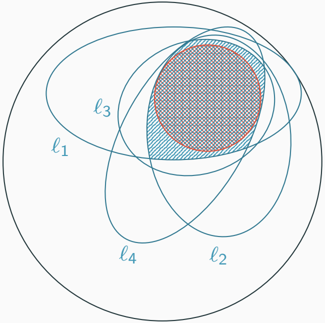

ℓ_1: 2416
ℓ_2: 2791
ℓ_3: 1961
ℓ_4: 2048

- **Overall scores** (tiers 1+2, w = 4)
  - $x \cdot s$ 490 $\cong$ 94% (19 lemmas)
  - $x \div s$ 396 $\cong$ 96% (36 lemmas)
  - $x \bmod s$ 400 $\cong$ 96% (15 lemmas)

Every Technique Has Scalability Issues

1,500 benchmarks* instantiated with bit-widths 16,...,8192, ∼ 85 sat, ∼ 1415 unsat

Solved Benchmarks

bw	**Bit-Blast** Bitwuzla	**Lazy+Layered** CVC4	**MCSAT** Yices2	**Int-Blast** cvc5	**PolySAT** Z3	**VBS**	**Bit-Blast+Abstr** Bitwuzla
16	1,495	1,458	1,394	1,116	696	**1,500**	
32	1,459	1,390	1,194	1,102	672	**1,490**	
64	1,440	1,368	1,112	1,077	668	**1,487**	
128	1,433	1,308	1,076	1,017	648	**1,488**	
256	1,388	1,232	987	916	637	**1,480**	
512	1,277	1,162	916	788	620	**1,421**	
1,024	1,065	774	794	613	608	**1,323**	
2,048	844	401	668	528	576	**1,133**	
4,096	816	300	572	428	562	**1,074**	
8,192	744	202	492	389	552	**993**	
	99% → 49%	97% → 13%	93% → 33%	74% → 26%	46% → 37%	**100% → 66%**	

Limits: 1,200 seconds, 8GB memory

* *syrew* benchmarks from [Niemetz et al. 2024]. 500 term and formula equivalence checks enumerated with cvc5's SyGuS solver using SyGuS grammar $\{0, 1, x, s, t, \approx, \not\approx, <_u, \leq_u, \sim, \&, \ll, \gg, \diamond\}$ for $\diamond \in \{\cdot, \div, \text{mod}\}$.

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4,096	816	300	572	428	562	**1,074**	**1,300**
8,192	744	202	492	389	552	**993**	**1,274**
	99% → 49%	97% → 13%	93% → 33%	74% → 26%	46% → 37%	**100% → 66%**	**99% → 85%**

Limits: 1,200 seconds, 8GB memory

100% → 88%

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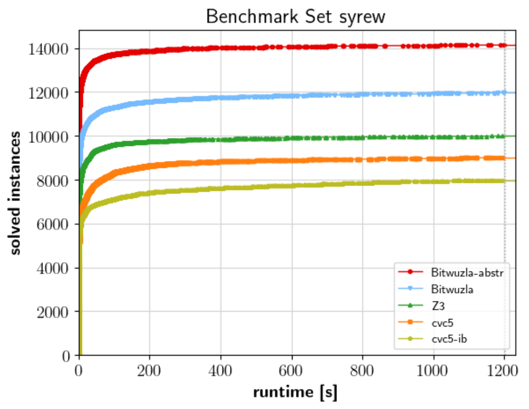
Evaluation

- **Configurations**
  - **Base Configuration**
    - Bitwuzla [0.3.2]
  - **Abstraction**
    - Bitwuzla-abstr [Bitwuzla + abstraction]
    - $\{\cdot, \div, \text{mod}\}$ of size ≥ 32
  - **Other Configurations**
    - cvc5 [1.1.0]
    - Z3 [4.12.4]
    - » support all logics supported by Bitwuzla
    - cvc5-ib [cvc5 int-blasting]
    - » translation from BV to NIA
    - » motivated by large bit-vectors
- **Setup**
  - Limits: 1200 seconds, 8 GB memory

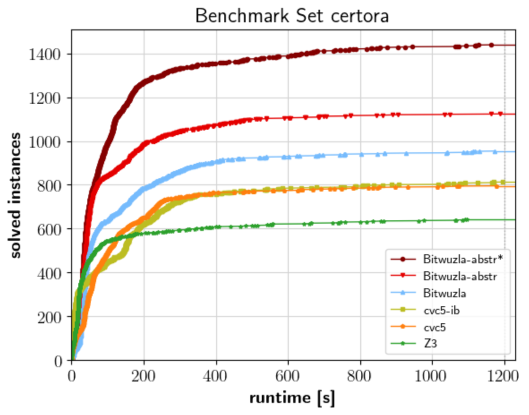
Evaluation

- **Crafted benchmarks**
  - *syrew*
    - **controlled set** to evaluate effectiveness
    - involving one of $\{\cdot, \div, \text{mod}\}$
      - 500 term and formula equivalence checks
      - enumerated by SyGuS (cvc5) for $w = 4$
      - instantiated for $w = \{16, 32, \ldots, 8192\}$
      - 1500 benchmarks, $\sim$ 85 sat, $\sim$ 1415 unsat



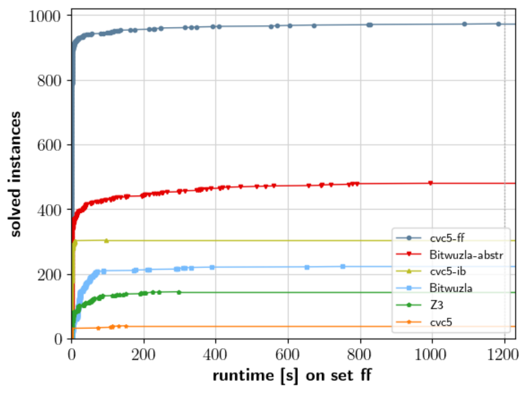
Evaluation

- **Smart contract verification**
  - » *certora* (Certora Prover)
  - » *ethereum* (hevm, Ethereum Foundation)
  - ▷ 256 bit bit-vectors
  - ▷ heavy use of $\{\cdot, \div, \text{mod}\}$



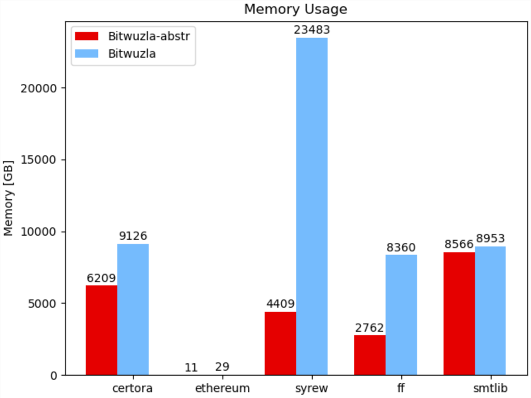
Evaluation

- ▶ **Translation validation of ZK proofs**
  - » *ff*
  - ▷ 2 sets: T_{FF} and T_{BV}
  - ▷ 510 bit bit-vectors
- ▷ **Additional configuration:**
  - cvc5-ff [cvc5 finite field solver]
  - on T_{FF} encoding of set *ff*



Evaluation: Memory Usage

- **SMT-LIB**
  - *smtlib*
    - all quantifier-free and quantified logics supported by Bitwuzla (24 in total)
    - combinations of BV, FP, UF, Arrays



Evaluation: FP logics

- **SMT-LIB**
  - » *smtlib*
  - all quantifier-free and quantified logics supported by Bitwuzla (24 in total)
  - combinations of BV, FP, UF, Arrays

- **FP Logics**
  - » Bitwuzla uses word-blasting for FP logics
    - Reduction of FP to bit-vectors with SymFPU [Brain et al. 2019b]
  - » Operations on unpacked formats require full precision arithmetic
    - Float32 `fp.mul`: 48-bit `bvmul`
    - Float64 `fp.mul`: 106-bit `bvmul`

Bitwuzla vs. Bitwuzla-abstr

Logic	Solved (Diff)	Time [s] (Common)
FP	+5	−16%
BVFP	0	−45%
QF_ABVFP	+1	−33%
QF_ABVFPLRA	0	−23%
QF_BVFP	+1	−45%
QF_BVFPLRA	+9	−46%
QF_FP	+23	−13%
QF_FPLRA	+1	−7%
QF_BV/float	+11	−16%

Evaluation: Abstraction Statistics over all Benchmark Sets

- **80% solved without bit-blasting of abstracted terms**
  - Tier 1-3 refinement lemmas were sufficent to solve the problem

Family	Solved	Abstr. Only	T1	T2	T3	T4
certora$_1$	255	**86.7%**	86.3%	78.0%	60.4%	13.3%
certora$_2$	395	**97.2%**	85.3%	46.3%	50.4%	2.8%
ethereum	24	**100%**	41.7%	29.2%	16.7%	0.0%
syrew	7,355	**91.9%**	98.1%	23.5%	12.3%	8.1%
ff	480	**95.2%**	84.6%	38.1%	34.8%	4.8%
smtlib	15,524	**73.9%**	64.3%	27.5%	36.5%	26.1%
Total	24,033	**80.4%**	75.6%	27.3%	29.5%	19.6%

- 20% required bit-blasting of abstracted terms
  - Only 37% of multiplication, 13% of division, 2% of remainder terms bit-blasted

- On average 37 refinement iterations (median 4)

Takeaways

- **Large bit-vectors** and/with **arithmetic challenging** for state-of-the-art approaches
  - arithmetic usually **in combination** with bitwise/shift/word operators
  - scalability **not only** an issue for bit-blasting
- Bit-Blasting still **best-performing** approach
  - alternative approaches **complementary**
- **Invertibility Conditions** powerful and versatile concept
- **CEGAR-style** abstraction-refinement for **theory BV** based on **bit-blasting**
  - **significantly improves scalability of bit-blasting**
  - **for majority of solved instances full arithmetic circuit not required**

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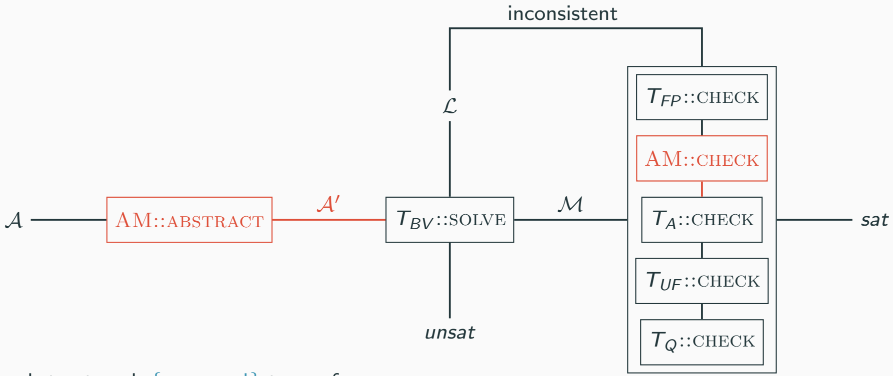
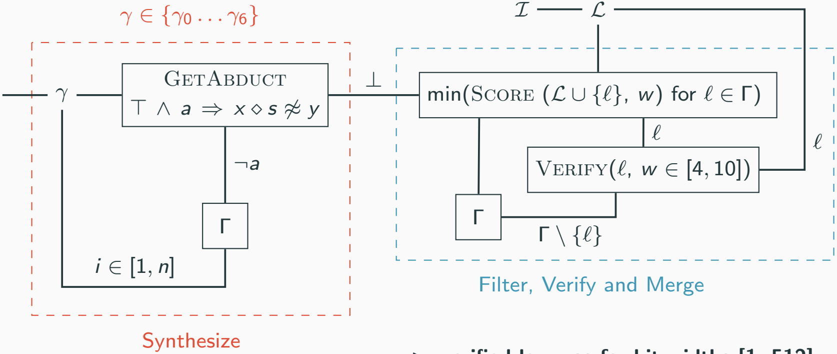
Bitwuzla: Lemmas on Demand Loop with Abstraction Module



- abstract each $\{\cdot, \div, \text{mod}\}$ term of size ≥ 32 with **fresh constant**
- optional: **assertion abstraction** [KrStr16]
  - interleaved with term abstraction
  - effective if unsat core very small
- order **not arbitrary**
  - T_{FP} word-blasted to T_{BV}
  - T_A, T_{UF} and T_Q require consistent T_{BV} abstraction

Lemma Synthesis



- **verified lemmas for bit-widths [1, 512]**
  - Bitwuzla, cvc5, Yices, Z3
  - 8 hours time limit, 8 GB memory limit
  - 16,896 benchmark, 6348 CPU hours

Evaluation

Benchmarks	Solver	Solved	Timeout	Memout	Time [s]	Mem [GB]
certora$_1$ (850)	ABSTR-TA	573	231	46	448k	2,492
	ABSTR-A	386	140	324	681k	5,201
	Bitwuzla-abstr	258	155	437	760k	4,807
	cvc5-ib	147	674	0	879k	667
	Bitwuzla	111	86	653	915k	6,182
	cvc5	90	113	610	923k	6,064
	Z3	30	447	373	989k	4,944
certora$_2$ (1,138)	ABSTR-TA	866	264	8	370k	1,024
	Bitwuzla-abstr	866	263	9	384k	1,402
	ABSTR-A	844	269	25	433k	2,661
	Bitwuzla	843	266	29	439k	2,944
	cvc5	705	223	210	603k	4,027
	cvc5-ib	666	472	0	643k	106
	Z3	612	492	34	679k	1,866
ethereum (3,173)	Bitwuzla-abstr	3,173	0	0	407	11
	Bitwuzla	3,173	0	0	720	29
	Z3	3,169	4	0	6k	107
	cvc5	3,158	0	1	18k	36
	cvc5-ib	3,141	20	0	39k	21

Evaluation

Benchmarks	Solver	Solved	Timeout	Memout	Time [s]	Mem [GB]
syrew (15,000)	Bitwuzla-abstr	14,142	583	276	1,225k	4,409
	Bitwuzla	11,961	744	2,296	3,955k	23,483
	Z3	9,992	833	4,175	6,198k	39,506
	cvc5	9,003	797	5,200	7,498k	48,421
	cvc5-ib	7,974	5,137	1,632	8,836k	19,850
ff (1,224)	cvc5-ff	973	129	122	313k	1,364
	Bitwuzla-abstr	480	729	15	913k	2,762
	cvc5-ib	304	822	98	1,104k	1,074
	Bitwuzla	223	71	930	1,211k	8,360
	Z3	145	56	1,023	1,299k	8,893
	cvc5	40	0	1,184	1,422k	9,523
smtlib (155,269)	Bitwuzla-abstr	148,554	1,944	152	8,770k	8,566
	Bitwuzla	148,492	1,966	193	8,748k	8,953
	Z3	145,121	4,846	565	13,528k	18,278
	cvc5	144,829	3,775	285	13,513k	11,029
	cvc5-ib	127,144	24,479	194	39,647k	15,233

36 unique (cvc5-ff and Bitwuzla-abstr on *ff*)

Limits: 1200 seconds CPU time limit, 8GB memory limit